

**Chapter 20 – Instruction Sheet**  
**Hypothesis Testing with proportions step by step**

STEP 1: Writing the Hypothesis

The null hypothesis:  $H_0$ , “H naught” states what the conventional or understood proportion is

ex) *In the 1950’s, only about 40% of high school graduates went on to college...*

$$H_0: p = 0.40$$

The null hypothesis is always an equals (=), setting it equal to what we assume the proportion to be.

The alternate hypothesis:  $H_A$  states what we wonder, what we are testing, how it compares to the null

ex) *...has the percentage of high school graduates changed?*

$$H_A: p \neq 0.40$$



Here are a few more examples:

a. *A governor is concerned about his “negatives”- the percentage of state residents who express disapproval of his job performance. His political committee pays for a series of TV ads, hoping that they can keep the negatives below 30%. They will use follow-up polling to assess the ad’s effectiveness.*

$$H_0: p = 0.30 \quad H_A: p < 0.30 \quad \text{**they are testing to see if they can get less than 30%}$$

b. *Only about 20% of people who try to quit smoking succeed. Sellers of a motivational tape claim that listening to the recorded messages can help people quit.*

$$H_0: p = 0.20 \quad H_A: p > 0.20 \quad \text{**they are testing to see if they can get more than 20%}$$

c. *Is a coin (normally 50% heads and 50% tails) actually fair?*

$$H_0: p = 0.50 \quad H_A: p \neq 0.50 \quad \text{**they don’t care if it is less or more than 50%, just that it is not 50%}$$

**Practice these. Write both the null and alternate hypothesis to test the following observations.**

1. In the 1980’s it was generally believed that congenital abnormalities affected about 5% of the nation’s children. Some people believe that the increase in the number of chemicals in the environment has led to an increase in the incidence of abnormalities. A recent study examined 384 children and found that 46 of them showed signs of an abnormality. Is there strong evidence that the risk has increased?

$$H_0: p = 0.05 \quad H_A: p > 0.05$$

2. In a recent year, 73% of first-year college students responding to a national survey identified “being very well-off financially” as an important personal goal. A state university finds that 132 of an SRS of 200 of its first-year students say that this goal is important. Is there good evidence that the proportion of all first-year students at this university who think being very well-off is important differs from the national value, 73%?

$$H_0: p = 0.73 \quad H_A: p \neq 0.73$$

3. A drug manufacturer claims that less than 10% of patients who take its new drug for treating Alzheimer’s disease will experience nausea. To test this claim, researchers conduct an experiment. They give the new drug to a random sample of 300 out of 5000 Alzheimer’s patients whose families have given informed consent for the patients to participate in the study. In all, 25 of the subjects experience nausea. Do you believe the drug manufacturer’s claim?

$$H_0: p = 0.10 \quad H_A: p < 0.10$$

## STEP 2: Determine Which Model to be Used

There are two parts to this step: check conditions & assumptions AND stating the model to be used.

### Conditions & Assumptions

These are the same as for confidence intervals:

- The Independence Assumption** which is checked through the randomization condition and the 10% condition
- Large Enough Sample Size Assumption** which is checked through the success/fail condition ( $\geq 10$  expected successes and  $\geq 10$  expected failures).

### Stating the Model

For now, we only have 1 model: the "one-proportion z-test." It is called this since we only have one proportion (the one we are testing) and we will be using a normal approximation or z-score to determine the probability.

Continuing the high school graduates example: ...To test this, a local university took a random sample of 2000 high school graduates...

"I can assume that these samples are independent because, as stated, a random sample was taken and 2000 is less than 10% of the entire population of high school graduates. I can also assume that the sample is large enough because there are more than 10 expected successes ( $2000 \times 0.40 = 800$ ) and more than 10 expected failures ( $2000 \times 0.60 = 1200$ ). Therefore I will be using a one-proportion z-test to test this assumption.

### Practice by testing conditions and stating the model for the previously listed practice problems.

1. In the 1980's it was generally believed that congenital abnormalities affected about 5% of the nation's children. Some people believe that the increase in the number of chemicals in the environment has led to an increase in the incidence of abnormalities. A recent study examined 384 children and found that 46 of them showed signs of an abnormality. Is there strong evidence that the risk has increased?

Random Sample  $\rightarrow$  not stated but assumed  
 $10\% \rightarrow 384 < 10\%$  of all children (population)  
Success/Fail (sample size)  $\rightarrow 0.05 \times 384 \geq 10$   
 $0.95 \times 384 \geq 10$

One  
Proportion  
z-test

2. In a recent year, 73% of first-year college students responding to a national survey identified "being very well-off financially" as an important personal goal. A state university finds that 132 of a SRS of 200 of its first-year students say that this goal is important. Is there good evidence that the proportion of all first-year students at this university who think being very well-off is important differs from the national value, 73%?

SRS-stated

$10\% \rightarrow 200 < 10\%$  of all first year students  
Success/Fail  $\rightarrow (0.73)(200) \geq 10$   
(Sample size)  $(0.27)(200) \geq 10$

All conditions are met,  
proceed with one  
proportion  
z-test

3. A drug manufacturer claims that less than 10% of patients who take its new drug for treating Alzheimer's disease will experience nausea. To test this claim, researchers conduct an experiment. They give the new drug to a random sample of 300 out of 5000 Alzheimer's patients whose families have given informed consent for the patients to participate in the study. In all, 25 of the subjects experience nausea. Do you believe the drug manufacturer's claim?

Random Sample stated

$10\% \rightarrow 300 < 10\%$  of 5000 Alzheimer patients  
Sample size  $\rightarrow (0.10)(300) \geq 10$   
 $(0.90)(300) \geq 10$

One proportion  
z-test

### STEP 3: Mechanics

There are 3 primary parts to this.

- 1: Calculate the standard deviation. We use the standard deviation, not the standard error, because the p being tested is an accepted population parameter, not a sample statistic.

\*We use the formula:  $\sqrt{\frac{pq}{n}}$ . Using the high school graduate example, the standard deviation is  $\sqrt{\frac{(0.40)(0.60)}{2000}} = 0.011$

**Practice – find the standard deviation for each of the previously listed practice problems.**

1. In the 1980's it was generally believed that congenital abnormalities affected about 5% of the nation's children. Some people believe that the increase in the number of chemicals in the environment has led to an increase in the incidence of abnormalities. A recent study examined 384 children and found that 46 of them showed signs of an abnormality. Is there strong evidence that the risk has increased?

$$\sqrt{\frac{(0.05)(0.95)}{384}} = 0.011$$

2. In a recent year, 73% of first-year college students responding to a national survey identified "being very well-off financially" as an important personal goal. A state university finds that 132 of an SRS of 200 of its first-year students say that this goal is important. Is there good evidence that the proportion of all first-year students at this university who think being very well-off is important differs from the national value, 73%?

$$0.031$$

3. A drug manufacturer claims that less than 10% of patients who take its new drug for treating Alzheimer's disease will experience nausea. To test this claim, researchers conduct an experiment. They give the new drug to a random sample of 300 out of 5000 Alzheimer's patients whose families have given informed consent for the patients to participate in the study. In all, 25 of the subjects experience nausea. Do you believe the drug manufacturer's claim?

$$0.017$$

- 2: Calculate the z-score for the observed proportion compared to the accepted population parameter.

\*Using the high school graduate example: ...out of the 2000 surveyed, 850 claimed to be going to college. So the observed proportion is:  $\hat{p} = \frac{850}{2000} = 0.425$  and to calculate the z-score:  $z = \frac{0.425 - 0.40}{0.011} = 2.273$

$$z = \frac{\hat{p} - p}{\sigma}$$

**Practice – find the z-score for the previously listed practice problems.**

1. In the 1980's it was generally believed that congenital abnormalities affected about 5% of the nation's children. Some people believe that the increase in the number of chemicals in the environment has led to an increase in the incidence of abnormalities. A recent study examined 384 children and found that 46 of them showed signs of an abnormality. Is there strong evidence that the risk has increased?

$$\hat{p} = \frac{46}{384} = 0.119 \quad z = \frac{0.119 - 0.05}{0.011} = 6.27$$

2. In a recent year, 73% of first-year college students responding to a national survey identified "being very well-off financially" as an important personal goal. A state university finds that 132 of an SRS of 200 of its first-year students say that this goal is important. Is there good evidence that the proportion of all first-year students at this university who think being very well-off is important differs from the national value, 73%?

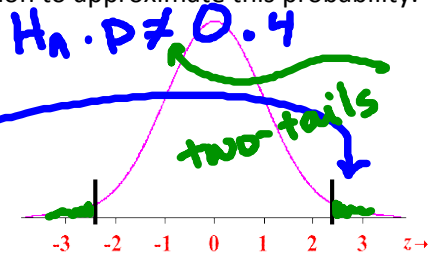
$$\hat{p} = \frac{132}{200} = 0.66 \quad z = \frac{0.66 - 0.73}{0.031} = -2.25$$

3. A drug manufacturer claims that less than 10% of patients who take its new drug for treating Alzheimer's disease will experience nausea. To test this claim, researchers conduct an experiment. They give the new drug to a random sample of 300 out of 5000 Alzheimer's patients whose families have given informed consent for the patients to participate in the study. In all, 25 of the subjects experience nausea. Do you believe the drug manufacturer's claim?

$$\hat{p} = \frac{25}{300} = 0.083 \quad z = \frac{(0.083 - 0.1)}{0.017} = -1.1$$

3: Calculate the probability of the observed proportion being true given that the population parameter is true – this is called the “p-value.” We use the z-table (or normalcdf) and the normal distribution to approximate this probability.

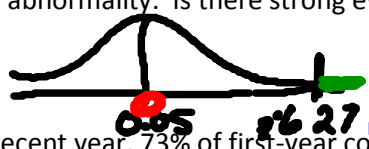
\*When looking at the high school graduate example, we see this is “two tailed” – since the  $H_A$  is  $\neq$  instead of  $<$  or  $>$ , the probability needs to cover both tails. Therefore we need to calculate both probabilities (both tails) and add them together. If this had been simply a  $<$  or  $>$ , we would have only needed to find the probability in one tail.



This can be seen in the picture. The probability of a z-score of 2.273 is 0.0116. So the entire probability is  $0.0116 \times 2 = 0.0232$ .

**Practice – find the p-value for the previously listed practice problems.**

1. In the 1980’s it was generally believed that congenital abnormalities affected about 5% of the nation’s children. Some people believe that the increase in the number of chemicals in the environment has led to an increase in the incidence of abnormalities. A recent study examined 384 children and found that 46 of them showed signs of an abnormality. Is there strong evidence that the risk has increased?



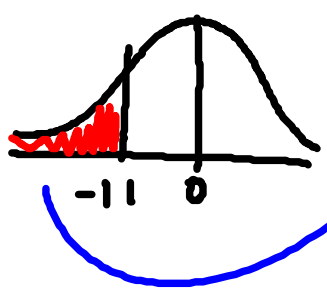
$H_A: p > 0.05$   
 normalcdf  
 Lbound -10  
 Ubound 9999  
 $\mu = 0$   
 $\sigma = 1$   
 p-value 0.00000000181

2. In a recent year, 73% of first-year college students responding to a national survey identified “being very well-off financially” as an important personal goal. A state university finds that 132 of an SRS of 200 of its first-year students say that this goal is important. Is there good evidence that the proportion of all first-year students at this university who think being very well-off is important differs from the national value, 73%?



$H_A: p \neq 0.73$   
 normalcdf (-999, -2.25, 0, 1)  
 $0.012 \times 2 = 0.024 = p\text{-val}$

3. A drug manufacturer claims that less than 10% of patients who take its new drug for treating Alzheimer’s disease will experience nausea. To test this claim, researchers conduct an experiment. They give the new drug to a random sample of 300 out of 5000 Alzheimer’s patients whose families have given informed consent for the patients to participate in the study. In all, 25 of the subjects experience nausea. Do you believe the drug manufacturer’s claim?



$H_A: p < 0.1$   
 $p\text{-value} = 0.158$

#### STEP 4: Conclusion

State a conclusion based on the calculated p-value with either a rejection of the null (if the p-value is large) or a "fail to reject" (wording is important – we never "accept" a null, we only "fail to reject" it). Usually 5% is a good place to make that decision, but not always.

\*To conclude our high school graduate example, a p-value of 0.0232 means that we reject the null. And stated in context, there is sufficient evidence to conclude that the 40% rate describing those who graduate from high school and go on to college is incorrect.

Practice – write conclusions for practice problems 1-3.

$$\begin{aligned} H_0: p = 0.05 \\ H_A: p > 0.05 \end{aligned}$$

1. In the 1980's it was generally believed that congenital abnormalities affected about 5% of the nation's children. Some people believe that the increase in the number of chemicals in the environment has led to an increase in the incidence of abnormalities. A recent study examined 384 children and found that 46 of them showed signs of an abnormality. Is there strong evidence that the risk has increased?

Due to a very low p-value of almost zero we reject the null. There is strong evidence that the risk of congenital abnormalities has increased.

$$\begin{aligned} H_0: p = 0.73 \\ H_A: p \neq 0.73 \end{aligned}$$

2. In a recent year, 73% of first-year college students responding to a national survey identified "being very well-off financially" as an important personal goal. A state university finds that 132 of an SRS of 200 of its first-year students say that this goal is important. Is there good evidence that the proportion of all first-year students at this university who think being very well-off is important differs from the national value, 73%?

Due to a low p-value of 0.024, we reject the null. There is good evidence that the proportion of all first year students at this university who...

$$\begin{aligned} H_0: p = 0.1 \\ H_A: p < 0.1 \end{aligned}$$

3. A drug manufacturer claims that less than 10% of patients who take its new drug for treating Alzheimer's disease will experience nausea. To test this claim, researchers conduct an experiment. They give the new drug to a random sample of 300 out of 5000 Alzheimer's patients whose families have given informed consent for the patients to participate in the study. In all, 25 of the subjects experience nausea. Do you believe the drug manufacturer's claim?

Due to a high p-value of 0.156, we fail to reject the null. There is not strong evidence

**One all together:**

In a group of 371 randomly selected University of Colorado students, 42 were left-handed. Is this significantly lower than the proportion of all Americans who are left-handed, which is 0.12?

**HYPOTHESIS:**

$$H_0: p = 0.12$$

$$H_A: p < 0.12$$

**DETERMINE THE MODEL:**

I assume the sample is independent since the group was randomly selected and 371 is less than 10% of the entire population of Americans. I also assume the sample size is large enough since both the expected number of successes (44.52) and the expected number of failures (326.48) is greater than or equal to 10. Since both assumptions are met, I will use a one-proportion z-test to determine.

**MECHANICS:**

$$\text{The standard deviation is } \sqrt{\frac{(0.12)(0.88)}{371}} = 0.0169$$

$$\text{The z-score is } \frac{\frac{42}{371} - 0.12}{0.0169} = -0.40$$

The probability of a z-score less than -0.40 is 0.3446 which is the p-value

**CONCLUSION:**

Due to a large p-value, I will fail to reject the null. There is not sufficient evidence that the group of University of Colorado students has a significantly lower proportion of left handers than that of all Americans.

**Homework: P 478 #19-22**