

AP Stats – Chapter 19

Standard Error

The true proportion of the population is represented by p , the problem is that often we do not know this, so the standard deviation: $\sqrt{\frac{pq}{n}}$ is only an approximation. So we often use standard error instead which is calculated from the observed proportion, \hat{p} and is calculated by the formula: $\sqrt{\frac{\hat{p}\hat{q}}{n}}$.

For the following problems, calculate or identify \hat{p} and the standard error, make sure to include labels.

1. A consumer group hoping to assess customer experiences with auto dealers surveys 167 people who recently bought new cars; 3% of them expressed dissatisfaction with the salesperson.

$\hat{p} = 0.03$ $\hat{q} = 0.97$ n

$$\sqrt{\frac{(0.03)(0.97)}{167}} = 0.0132 = \text{SE}$$

Standard Error

2. What percent of college students have cell phones? 2883 students were asked as they entered a football stadium, and 243 said they had phones with them.

$\hat{p} = \frac{243}{2883} = 0.084$

$\hat{q} = \frac{2640}{2883} = 0.916$

$$\text{SE} = \sqrt{\frac{(0.084)(0.916)}{2883}} = 0.005$$

3. 240 potato plants in a field in Main are randomly checked, and only 7 show signs of blight. How severe is the blight problem for the US potato industry?

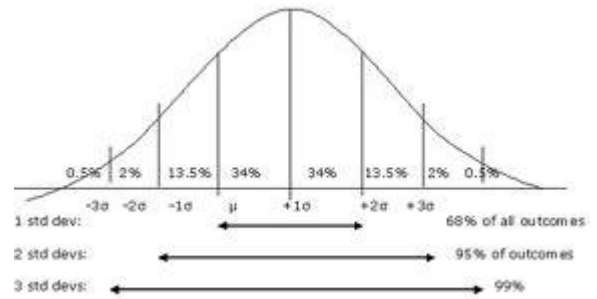
$$\text{SE} = 0.010$$

4. 12 of the 309 employees of a small company suffered an injury on the job last year. What can the company expect in future years?

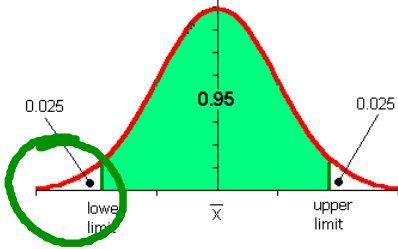
$$\text{SE} = 0.010$$

Confidence intervals

A confidence interval essentially expresses an interval and how confident we are that the true (population) proportion falls within that interval. In other words, if a 95% confidence interval is 0.341 and 0.427, we are confident that 95% of all random samples will yield intervals that include the true proportion. We use our normal distribution to approximate these as illustrated in the picture.



To calculate a confidence interval, we need to first calculate a critical value, or a critical z, notated by z^* . This is the z-score that places us in the center of the confidence interval. One of the most common is for 95% confidence:



Since the middle 95% leaves 2.5% or 0.025 on either side, we find the z-score for 0.025 with either the table or with the calculator using the command `invNorm(0.025)`. The critical z for 95% confidence is **1.96**. ← 95%

z	.00	.01	.02	.03	.04	.05	.06
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314

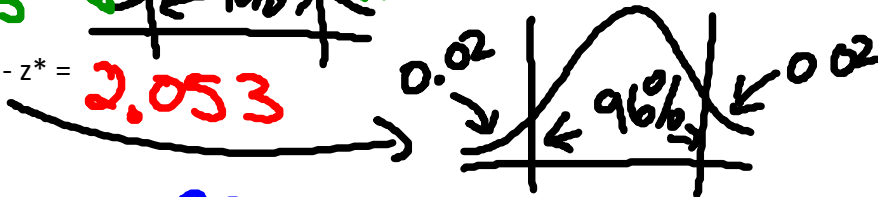
Calculate the critical z for the following common confidence intervals (these would be EXCELLENT to memorize):

1. 90% - $z^* = 1.645$



calculator \Rightarrow `invNorm(.05)`
 $= -1.645$

2. 96% - $z^* = 2.053$



`invNorm(0.02)`
 $= -2.053$

3. 98% - $z^* = 2.326$

4. 99.7% - $z^* = 2.967$

Margin of error

In a confidence interval the center, or mean, of the interval is \hat{p} , the observed proportion. The margin of error is the interval on either side of \hat{p} . It is calculated by multiplying the critical z by the standard error. This is represented by the formulas:

$$ME = z * SE \text{ or } ME = z * \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

Calculate the margin of error using the following problems and the given confidence intervals:

1. 95% confidence: A consumer group hoping to assess customer experiences with auto dealers surveys 167 people who recently bought new cars; 3% of them expressed dissatisfaction with the salesperson.

$$ME = z^* \cdot SE \\ = (1.96)(0.0132) = 0.0259$$

2. 90% confidence: What percent of college students have cell phones? 2883 students were asked as they entered a football stadium, and 243 said they had phones with them.

$$ME = 0.0082$$

3. 96% confidence: 240 potato plants in a field in Main are randomly checked, and only 7 show signs of blight. How severe is the blight problem for the US potato industry?

$$ME = 0.020$$

4. 98% confidence: 12 of the 309 employees of a small company suffered an injury on the job last year. What can the company expect in future years?

$$ME = 0.023$$

Confidence Interval:

The confidence interval is calculated by adding and subtracting the margin of error from the observed proportion, forming an interval in which the true proportion is most likely located.

For example let us construct a 95% confidence interval for the following. In January 2007 *Consumer Reports* published their study of bacterial contamination of chicken sold in the United States. They purchased 525 broiler chickens from various kinds of food stores in 23 states and tested them for types of bacteria that cause food-borne illnesses. Laboratory results indicated that 83% of these chickens were infected with *campylobacter*.

Step 1: Calculate the standard error:

$$SE = \sqrt{\frac{\hat{p}\hat{q}}{n}} = \sqrt{\frac{0.83 \times 0.17}{525}} = 0.0164$$

Step 2: Determine the critical z (z*):

for 95% confidence the critical z is 1.96 as calculated earlier

Step 3: Calculate the margin of error:

$$ME = z^* \times SE = 1.96 \times 0.0164 = 0.0321$$

Step 4: Add and subtract the margin of error to and from the observed proportion (\hat{p}):

$$0.83 - 0.0321 = 0.7979$$

$$0.83 + 0.0321 = 0.8621$$

Conclusion:

We are 95% confident that the true proportion of chickens infected with *campylobacter* is between 0.798 and 0.862.

Calculate the following confidence intervals:

1. A consumer group hoping to assess customer experiences with auto dealers surveys 167 people who recently bought new cars. 3% of them expressed dissatisfaction with the salesperson. Construct a 95% confidence interval to contain the true proportion of customers who are dissatisfied with the salesperson.

$$0.03 - 0.025 = 0.005$$

$$0.03 + 0.025 = 0.055$$

We are 95% confident that the true population proportion of customers who are dissatisfied

2. What percent of college students have cell phones? 2883 students were asked as they entered a football stadium, and 243 said they had phones with them. Construct a 90% confidence interval to estimate the true proportion of students with cell phones.

is between 0.005 and 0.055.

3. 240 potato plants in a field in Main are randomly checked, and only 7 show signs of blight. How severe is the blight problem for the US potato industry? Construct a 96% confidence interval to describe this problem.

4. 12 of the 309 employees of a small company suffered an injury on the job last year. Construct a 98% confidence interval of what can the company expect in future years?