Friday, March 15, 2019

- Warm-up
  - In the 1986-87 regular season, Magic Johnson of the Los Angeles Lakers led the NBA in assists. His average over 80 games was 12.2 assists per game, with a standard deviation of 3.96 assists per game. Create and interpret a 95% confidence interval for the average number of assists per game of the regular season.
- Check homework
- More about t-distributions – hypothesis tests

Warm-up

$df = 79$ $\quad t^*_{79} \approx 1.99$

In the 1986-87 regular season, Magic Johnson of the Los Angeles Lakers led the NBA in assists. His average over 80 games was 12.2 assists per game, with a standard deviation of 3.96 assists per game. Create and interpret a 95% confidence interval for the average number of assists per game of the regular season.

Representative

80 < 10% of games

80 > 30 assume nearly normal due to CLT

1 sample t-interval

$$12.2 \pm 1.99 \frac{3.96}{\sqrt{80}}$$

$$12.2 \pm 0.881$$

$$(11.31, 13.08)$$

We are 95% confident that the average number of assists per game in a regular season is between 11.31 and 13.08.

Objectives

Content Objectives: I will begin to work with t-values.

Social Objective: I will listen to others and be respectful.

Language Objective: I will take clear notes that I will be able to refer to.

One-sample t-test for the mean

- The conditions for the one-sample *t*-test for the mean are the same as for the one-sample *t*-interval.
- We test the hypothesis $H_0: \mu = \mu_0$ using the statistic

$$t_{n-1} = \frac{\bar{y} - \mu_0}{SE(\bar{y})}$$

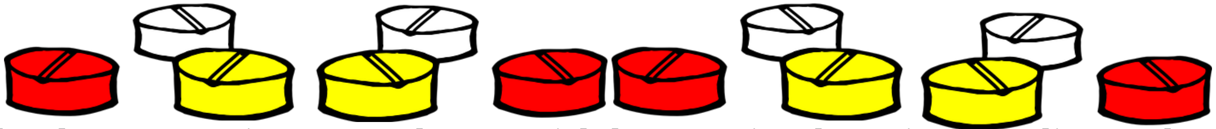
- The standard error of the sample mean is $SE(\bar{y}) = \frac{s}{\sqrt{n}}$

- When the conditions are met and the null hypothesis is true, this statistic follows a Student's *t* model with $n - 1$ *df.* We use that model to obtain a P-value.

What Conditions?

- Same as for confidence intervals…

Practice Problem



- A drug manufacturer forms tablets by compressing a granular material that contains the active ingredient and various filers. The hardness of a sample from each batch of tablets produced is measured to control the compression process. The target value for the hardness is $\mu = 11.5$. the hardness data for a random sample of 20 tablets are:

11.627	11.613	11.493	11.602	11.360
11.374	11.592	11.458	11.552	11.463
11.383	11.715	11.485	11.509	11.429
11.477	11.570	11.623	11.472	11.531

$H_0: \mu = 11.5$

$H_a: \mu \neq 11.5$

Random sample stated

20 < 10% all pills made

box plot shows fairly symmetric with no outliers : nearly normal

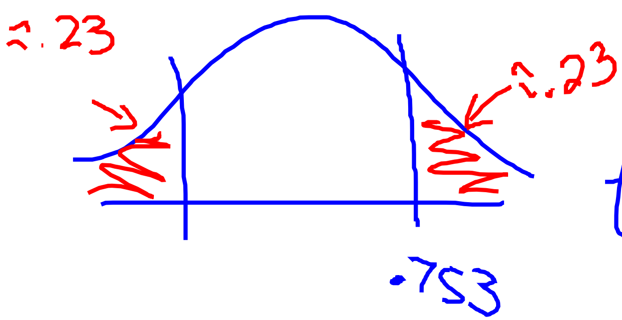
- Is there significant evidence at the 5% level that the mean hardness of the tablets differs from the target value?

1 sample t-test

$t_{19} = \dfrac{(11.516 - 11.5)}{\left(\dfrac{0.095}{\sqrt{20}}\right)}$

$t_{19} = 0.753$

p value ≈ 0.46



Due to a very large p value of about 0.46, we fail to reject the null. There is not significant evidence that the mean hardness differs from the target value.

Errors?

Remember that "statistically significant" does not mean "actually important" or "meaningful."

Because of this, it's always a good idea when we test a hypothesis to check the confidence interval and think about likely values for the mean.

Confidence intervals and hypothesis tests are built from the same calculations.

In fact, they are complementary ways of looking at the same question.

The confidence interval contains all the null hypothesis values we can't reject with these data.

More precisely, a level *C* confidence interval contains *all* of the plausible null hypothesis values that would *not* be rejected by a two-sided hypothesis text at alpha level 1 – *C*.

So a 95% confidence interval matches a 0.05 level two-sided test for these data.

Confidence intervals are naturally two-sided, so they match exactly with two-sided hypothesis tests.

When the hypothesis is one sided, the corresponding alpha level is (1 – *C*)/2.

Sample Size

- To find the sample size needed for a particular confidence level with a particular margin of error (*ME*), solve this equation for *n* :

$$ME = t^*_{n-1} \frac{s}{\sqrt{n}}$$

- The problem with using the equation above is that we don't know most of the values. We can overcome this:
  - We can use *s* from a small pilot study.
  - We can use *z** in place of the necessary *t* value.

- **Sample size calculations are *never* exact.**
  - **The margin of error you find *after* collecting the data won't match exactly the one you used to find *n*.**

- **The sample size formula depends on quantities you won't have until you collect the data, but using it is an important first step.**
  - **Before you collect data, it's always a good idea to know whether the sample size is large enough to give you a good chance of being able to tell you what you want to know.**

$$ME = t^{*}_{n-1}\frac{s}{\sqrt{n}}$$



What about Degrees of Freedom?

- If only we knew the true population mean, *μ*, we would find the sample standard deviation as

$$s = \sqrt{\frac{\sum (y - \mu)^2}{n}}.$$

- But, we use $\bar{y}$ instead of *μ*, though, and that causes a problem.

- When we use $\sum\left(y - \bar{y}\right)^2$ instead of $\sum\left(y - \mu\right)^2$ to calculate *s*, our standard deviation estimate would be too small.

What about Degrees of Freedom?

The amazing mathematical fact is that we can compensate for the smaller sum exactly by dividing by $n - 1$ which we call the degrees of freedom.

$$s = \sqrt{\frac{\Sigma(y - \bar{y})^2}{n - 1}}$$

What Can Go Wrong?

Don't confuse proportions and means.

Ways to Not Be Normal:

- **Beware of multimodality.**
  - **The Nearly Normal Condition clearly fails if a histogram of the data has two or more modes.**
- **Beware of skewed data.**
  - **If the data are very skewed, try re-expressing the variable.**
- **Set outliers aside—but remember to report on these outliers individually.**

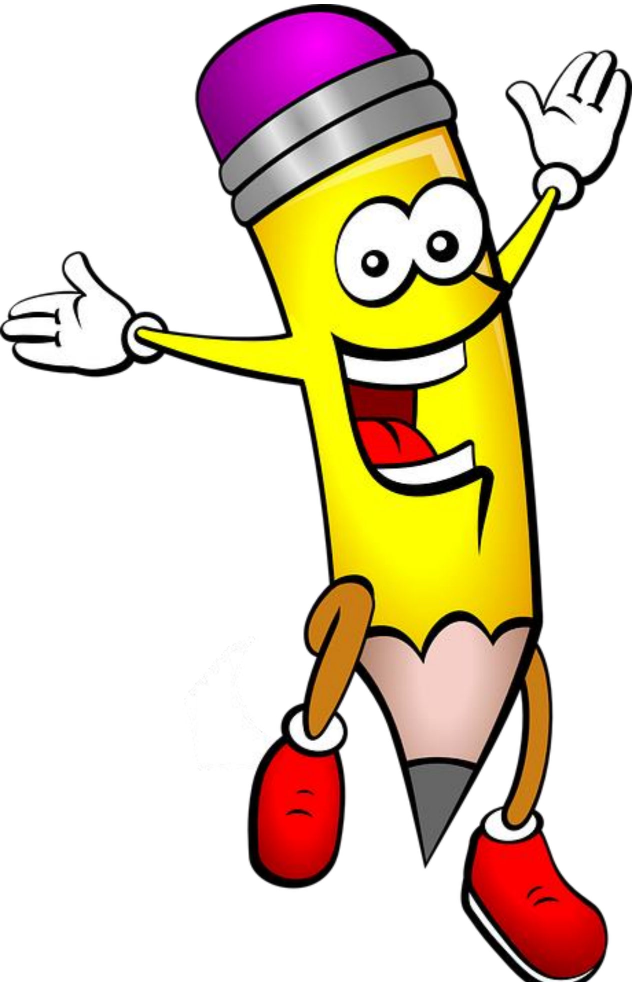
…And of Course:

- **Watch out for bias—we can never overcome the problems of a biased sample.**
- **Make sure data are independent.**
  - **Check for random sampling and the 10% Condition.**
- **Make sure that data are from an appropriately randomized sample.**

...And of Course, again:

- Interpret your confidence interval correctly.
  - Many statements that sound tempting are, in fact, misinterpretations of a confidence interval for a mean.
  - A confidence interval is about the mean of the population, not about the means of samples, individuals in samples, or individuals in the population.

What have we learned?



- Statistical inference for means relies on the same concepts as for proportions—only the mechanics and the model have changed.
  - What we say about a population mean is inferred from the data.
  - Student's *t* family based on degrees of freedom.
  - Ruler for measuring variability is SE.
  - Find ME based on that ruler and a student's t model.
  - Use that ruler to test hypotheses about the population mean.

Homework

- Pg 557 (29, 30)