

2019 AP[®] STATISTICS FREE-RESPONSE QUESTIONS

STATISTICS

SECTION II

Part A

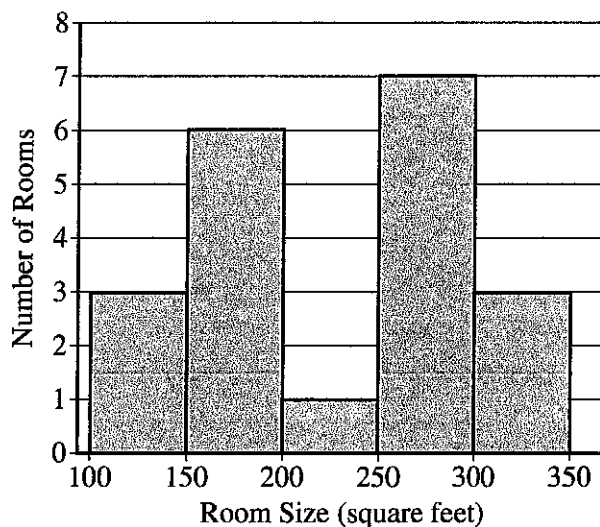
Questions 1-5

Spent about 1 hour and 5 minutes on this part of the exam.

Percent of Section II score—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. The sizes, in square feet, of the 20 rooms in a student residence hall at a certain university are summarized in the following histogram.

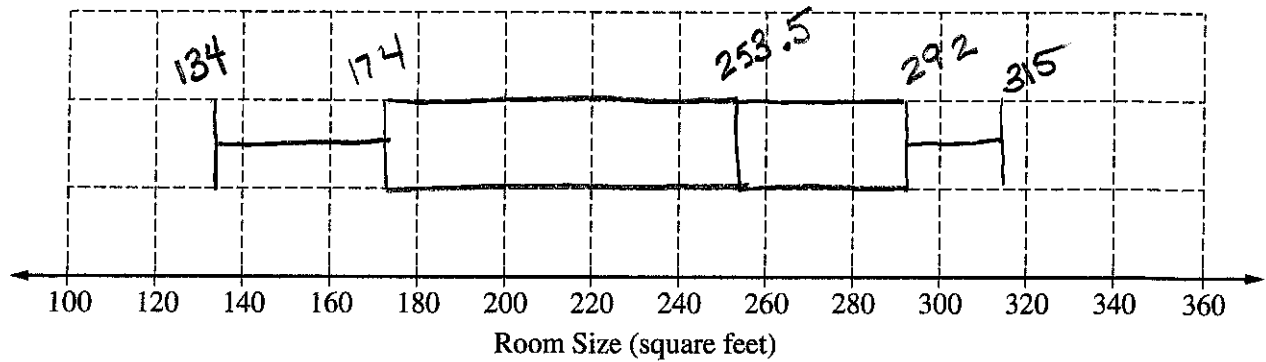


- (a) Based on the histogram, write a few sentences describing the distribution of room size in the residence hall.
- (b) Summary statistics for the sizes are given in the following table.

Mean	Standard Deviation	Min	Q1	Median	Q3	Max
231.4	68.12	134	174	253.5	292	315

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Determine whether there are potential outliers in the data. Then use the following grid to sketch a boxplot of room size.



- (c) What characteristic of the shape of the distribution of room size is apparent from the histogram but not from the boxplot?

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2. Researchers are investigating the effectiveness of using a fungus to control the spread of an insect that destroys trees. The researchers will create four different concentrations of fungus mixtures: 0 milliliters per liter (ml/L), 1.25 ml/L, 2.5 ml/L, and 3.75 ml/L. An equal number of the insects will be placed into 20 individual containers. The group of insects in each container will be sprayed with one of the four mixtures, and the researchers will record the number of insects that are still alive in each container one week after spraying.

(a) Identify the treatments, experimental units, and response variable of the experiment.

Treatments:

Experimental units:

Response variable:

(b) Does the experiment have a control group? Explain your answer.

(c) Describe how the treatments can be randomly assigned to the experimental units so that each treatment has the same number of units.

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3. A medical researcher surveyed a large group of men and women about whether they take medicine as prescribed. The responses were categorized as never, sometimes, or always. The relative frequency of each category is shown in the table.

	Never	Sometimes	Always	Total
Men	0.0564	0.2016	0.2120	0.4700
Women	0.0636	0.1384	0.3280	0.5300
Total	0.1200	0.3400	0.5400	1.0000

- (a) One person from those surveyed will be selected at random.
- What is the probability that the person selected will be someone whose response is never and who is a woman?
 - What is the probability that the person selected will be someone whose response is never or who is a woman?
 - What is the probability that the person selected will be someone whose response is never given that the person is a woman?
- (b) For the people surveyed, are the events of being a person whose response is never and being a woman independent? Justify your answer.
- (c) Assume that, in a large population, the probability that a person will always take medicine as prescribed is 0.54. If 5 people are selected at random from the population, what is the probability that at least 4 of the people selected will always take medicine as prescribed? Support your answer.

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4. Tumbleweed, commonly found in the western United States, is the dried structure of certain plants that are blown by the wind. Kochia, a type of plant that turns into tumbleweed at the end of the summer, is a problem for farmers because it takes nutrients away from soil that would otherwise go to more beneficial plants. Scientists are concerned that kochia plants are becoming resistant to the most commonly used herbicide, glyphosate. In 2014, 19.7 percent of 61 randomly selected kochia plants were resistant to glyphosate. In 2017, 38.5 percent of 52 randomly selected kochia plants were resistant to glyphosate. Do the data provide convincing statistical evidence, at the level of $\alpha = 0.05$, that there has been an increase in the proportion of all kochia plants that are resistant to glyphosate?

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5. A company that manufactures smartphones developed a new battery that has a longer life span than that of a traditional battery. From the date of purchase of a smartphone, the distribution of the life span of the new battery is approximately normal with mean 30 months and standard deviation 8 months. For the price of \$50, the company offers a two-year warranty on the new battery for customers who purchase a smartphone. The warranty guarantees that the smartphone will be replaced at no cost to the customer if the battery no longer works within 24 months from the date of purchase.
- (a) In how many months from the date of purchase is it expected that 25 percent of the batteries will no longer work? Justify your answer.
- (b) Suppose one customer who purchases the warranty is selected at random. What is the probability that the customer selected will require a replacement within 24 months from the date of purchase because the battery no longer works?
- (c) The company has a gain of \$50 for each customer who purchases a warranty but does not require a replacement. The company has a loss (negative gain) of \$150 for each customer who purchases a warranty and does require a replacement. What is the expected value of the gain for the company for each warranty purchased?

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STATISTICS

SECTION II

Part B

Question 6

Spend about 25 minutes on this part of the exam.

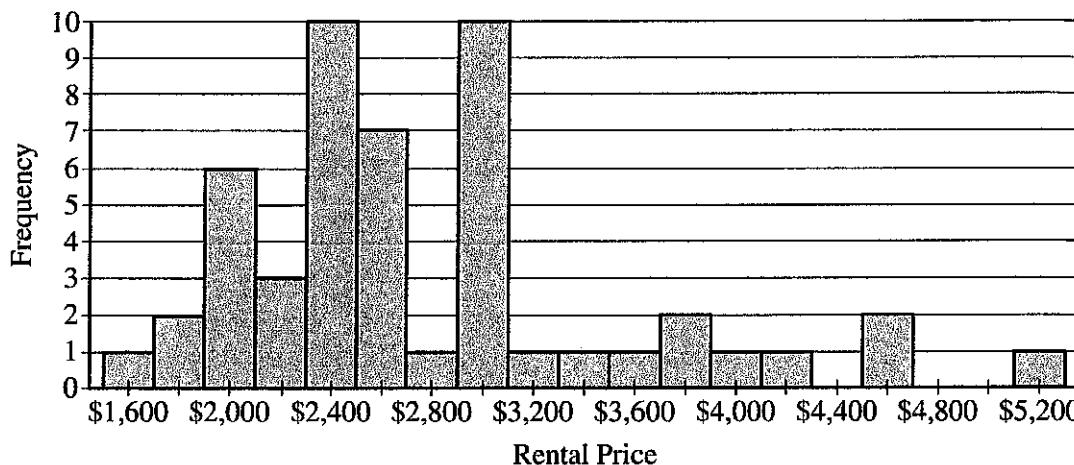
Percent of Section II score—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. Emma is moving to a large city and is investigating typical monthly rental prices of available one-bedroom apartments. She obtained a random sample of rental prices for 50 one-bedroom apartments taken from a Web site where people voluntarily list available apartments.

(a) Describe the population for which it is appropriate for Emma to generalize the results from her sample.

The distribution of the 50 rental prices of the available apartments is shown in the following histogram.



- (b) Emma wants to estimate the typical rental price of a one-bedroom apartment in the city. Based on the distribution shown, what is a disadvantage of using the mean rather than the median as an estimate of the typical rental price?
- (c) Instead of using the sample median as the point estimate for the population median, Emma wants to use an interval estimate. However, computing an interval estimate requires knowing the sampling distribution of the sample median for samples of size 50. Emma has one point, her sample median, in that sampling distribution. Using information about rental prices that are available on the Web site, describe how someone could develop a theoretical sampling distribution of the sample median for samples of size 50.

Because Emma does not have the resources to develop the theoretical sampling distribution, she estimates the sampling distribution of the sample median using a process called bootstrapping. In the bootstrapping process, a computer program performs the following steps.

- Take a random sample, with replacement, of size 50 from the original sample.
- Calculate and record the median of the sample.
- Repeat the process to obtain a total of 15,000 medians.

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Emma ran the bootstrap process, and the following frequency table is the bootstrap distribution showing her results of generating 15,000 medians.

Bootstrap Distribution of Medians					
Median	Frequency	Median	Frequency	Median	Frequency
2,345	1	2,585	1	2,825	247
2,390	13	2,587.5	171	2,837.5	7
2,395	18	2,600	22	2,847.5	1
2,400	56	2,612.5	1,190	2,872.5	317
88 — 2,445	4	2,625	174	2,885	10
2,447.5	56	2,672.5	5	2,950	700
203 — 2,450	55	2,675	1,924	2,962.5	93
2,475	3	2,687.5	1,341	2,972.5	6
2,495	66	2,700	2,825	2,975	65
408 — 2,497.5	136	2,735	35	2,985	12
2307 — 2,500	1,899	2,747.5	619	2,987.5	1
2,522.5	2	2,750	2	2,995	6
2,525	945	2,795	278	3,000	2
2,550	1,673	2,812.5	16	3,062.5	3

The bootstrap distribution provides an approximation of the sampling distribution of the sample median. A confidence interval for the median can be constructed using a percentage of the values in the middle of the bootstrap distribution.

- (d) Use the frequency table to find the following.
- (i) Value of the 5th percentile:
 - (ii) Value of the 95th percentile:
- (e) Find the percentage of bootstrap medians in the table that are equal to or between the values found in part (d).
- (f) Use your values from parts (d) and (e) to construct and interpret a confidence interval for the median rental price.

STOP

END OF EXAM

AP Stats 2019

1) a) The distribution of room size in the residence hall is centered between 200 and 250 square feet, has a range between 150 and 250 feet, is fairly symmetric and bimodal.

b) $IQR = 292 - 174$
 $= 118$

lower fence $= 174 - 1.5(118)$
 $= 174 - 177 = -3$ (no)

upper fence $= 292 + 1.5(118)$
 $= 292 + 177 = 469$ (max = 315)

no outliers

c) The peaks (2 modes) between 150 and 200 square feet and 250 and 300 square feet can be seen in the histogram, but not on the box plot.

2) a) Treatments: concentrations of fungus mixtures

0 ml/L

1.25 ml/L

2.5 ml/L

3.75 ml/L

Experimental units: groups of insects in each container

Response variable: the number of insects alive in each container one week after spraying

b) Yes - the control group is the group receiving 0 ml/L of fungus mixture. 0 ml/L would be no fungus and would provide a baseline.

2)

c) The numbers 1-20 could be put in a hat (each number assigned to a particular container of insects). 5 numbers can be drawn per treatment to assign a container to that treatment

3) a) i) $P(\text{Never AND Woman}) = 0.0636$
ii) $P(\text{Never OR Woman}) = 0.1200 + 0.5300 - 0.0636$
 $= 0.5864$
iii) $P(\text{Never} | \text{Woman}) = \frac{P(N \cap W)}{P(W)}$
 $= \frac{0.0636}{0.5300} = 0.12$

b) $P(N|W) \stackrel{?}{=} P(N)$
 $0.12 \neq 0.12$

Yes, being a woman is independent of never taking medicine because the probability of never given being a woman is the same as the probability of just being a woman therefore the condition of being a woman does not change the probability of never taking medicine as prescribed.

c) $p = 0.54$
 $n = 5$

lower bound = 4

upper bound = 5

binomial distribution

0.2415

4) 2014 \rightarrow 19.7% of 61 were resistant
 2017 \rightarrow 38.5% of 52 were resistant
 $\alpha = 0.05$

$H_0: p_{14} = p_{17}$
 $H_A: p_{14} < p_{17}$

2014
 "randomly selected"
 61 < 10% of all Kochia
 $(19.7\%)(61) = 12.017 > 10$
 $(80.3\%)(61) = 48.983 > 10$

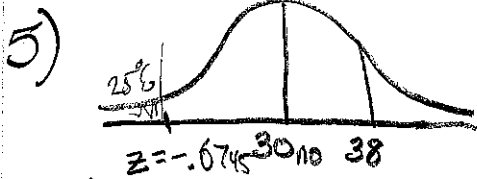
2017
 "randomly selected"
 52 < 10% of ^{all} Kochia
 $(38.5\%)(52) = 20.02 > 10$
 $(61.5\%)(52) = 31.98 > 10$

independent groups
 since randomly selected

2 prop z test
 $z = -2.2095$
 p-value = 0.0136



Due to a p-value of 0.0136 which is lower than the α of 0.05, we reject the null. There is sufficient statistical evidence that there has been an increase in the proportion of all Kochia plants that are resistant to glyphosate



\$50 = 2 yr. warranty (24 mo)

a) $\text{invNorm}(0.25) = -0.6745$
 $-0.6745 = \frac{x - 30}{8}$

$x = 24.6041$

25% of the batteries will no longer work after about 25 months

$$5 \text{ b) } P(b < 24) \Rightarrow P(z < -0.75) = 0.2266$$

$$z = \frac{24 - 30}{8}$$

$$= -0.75$$

22.66% chance
the battery will need replacement
within 24 mo of purchase

$$c) P(\text{no repl.}) * 50 + P(\text{repla.}) * -150$$

$$P(b > 24) * 50 \quad P(b \leq 24) * -150$$

$$0.7734 * 50 + 0.2266 * -150$$

$$\$4.68$$

expected value of the
gain for each warranty
purchased

6 a) Because these apartments were voluntarily listed and are available - the population is available one bedroom apartments being posted for rent on that website.

b) The right skew in the distribution would make the mean rental price, which is a mathematical average, higher than the median, which is simply the middle value. This higher estimate of the mean would cause her to potentially overestimate her rent, and be afraid she couldn't afford the apartment.

c) I would create an interval similar to one for means by using the point estimate of the median $\pm \text{IQR} / \sqrt{50}$. I would use the IQR because it is resistant to outliers like the median as opposed to the standard deviation which is not

6
d) i) 5th %ile $\rightarrow 0.05 \times 15,000 = 750$ th place
\$ 2500

ii) 95th % ile $\rightarrow 0.95 \times 15,000 = 14,250$ th place
\$ 2950 (or 750 from top)

e) 408 are less $15,000 - 408 = 14,592$
188 are more $14,592 - 188 = 14,404$ boundaries

$$\frac{14404}{15000} =$$

96.03%
are equal to or between

f) (2500, 2950) \rightarrow 96% Confidence Interval
I am 96% confident that the true median rent of a one bedroom apartment in this large city is between \$2500 and \$2950 per month.