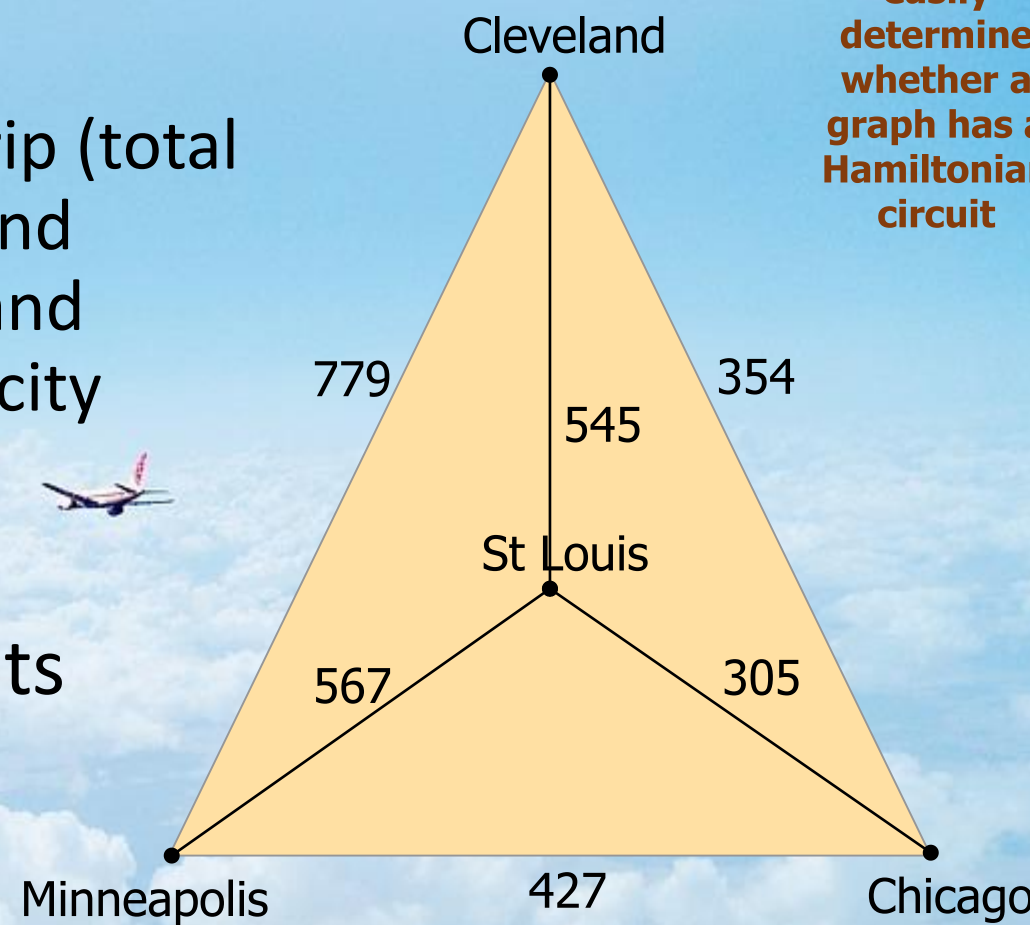
Wednesday, September 7, 2016

Unlike Euler circuits, no method has been found to easily determine whether a graph has a Hamiltonian circuit

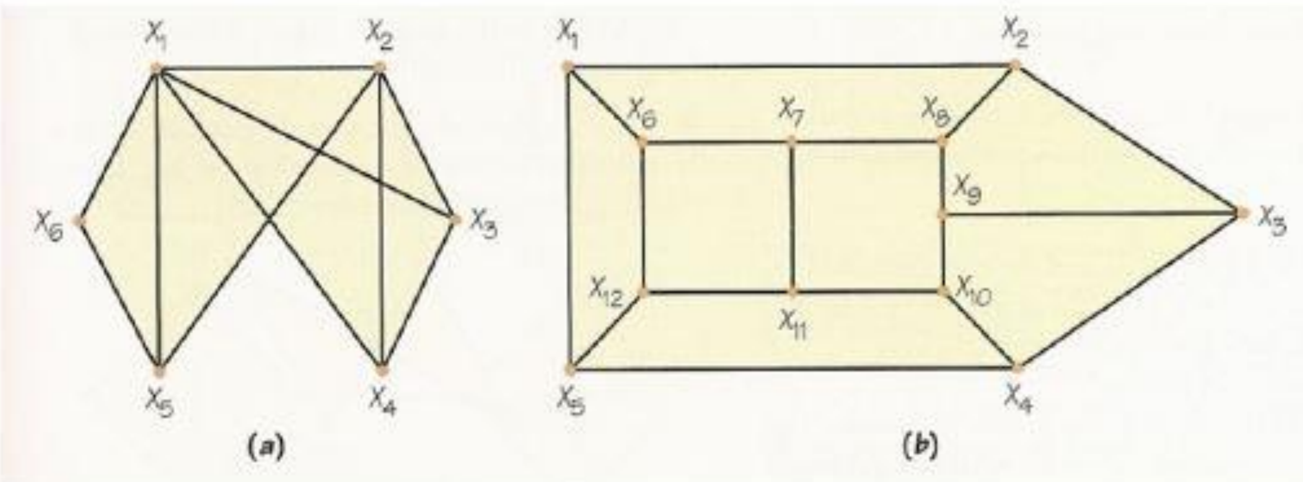
- Warm-up
  - Find the shortest trip (total distance) starting and ending in Chicago and visiting each other city once.
- Minimum-cost Hamiltonian Circuits
- Practice
- Homework time

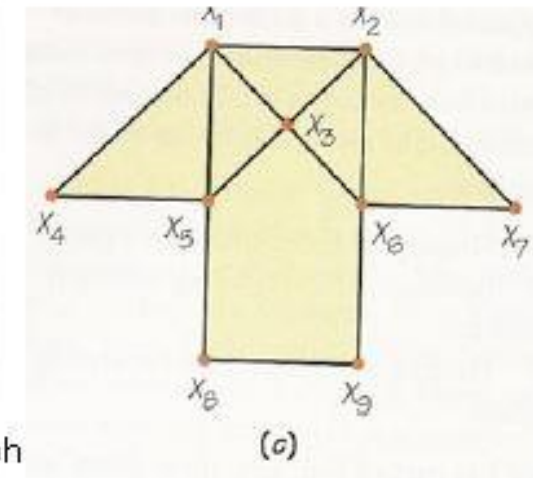


Check Homework

Homework 2.1

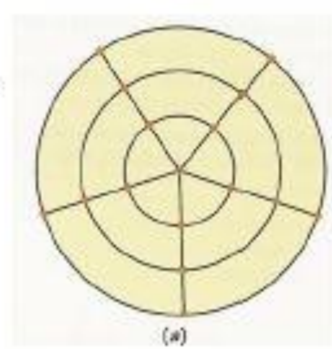
1. For each graph below write a Hamiltonian circuit starting at X_1

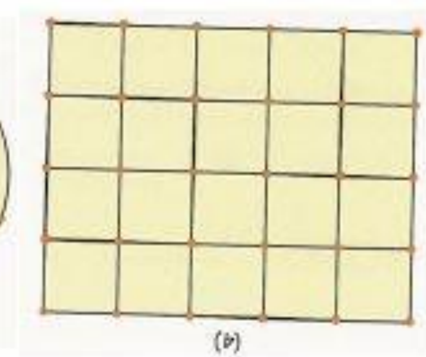




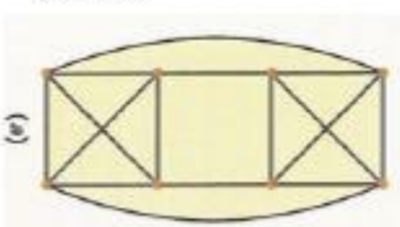
3. If the vertex X_1 and the edges attached to X_1 are removed from the graphs in #1, do the new graphs that result still have Hamiltonian circuits?

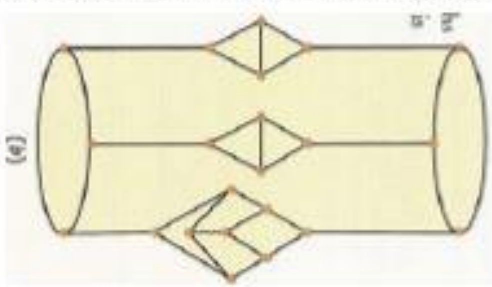
4. For each of the graphs, add edges to indicate a Hamiltonian circuit.





5. For each of the graphs below, determine if there is a Hamiltonian circuit.





Objectives

- Content Objective: Apply the Fundamental Principal of Counting to the Traveling Salesman Problem.
- Social Objective: Listen well to teacher and classmates.
- Language Objective: Clearly write the Brute Force algorithm, the Nearest Neighbor algorithm, and the Sorted Edges algorithm for solving the Traveling Salesman Problem.

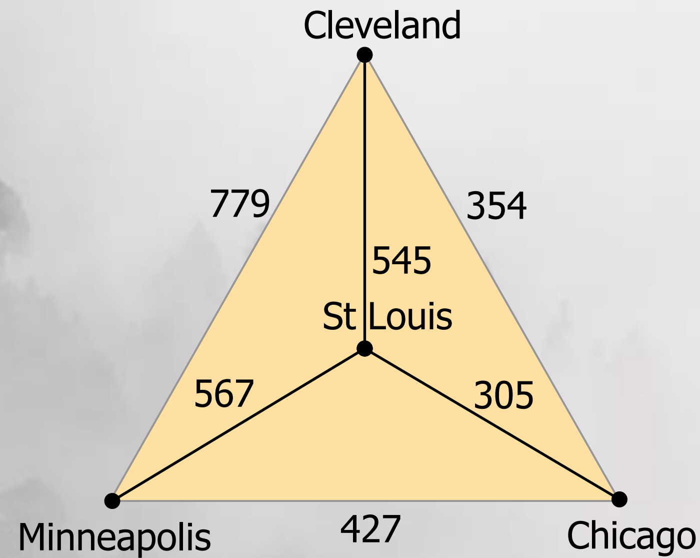
Minimum-Cost Hamiltonian Circuit

Warm-up

Algorithm (step-by-step process)

Brute Force Algorithm

- Generate all possible Hamiltonian circuits.
- Add up the **weights** (distances) on the edges of each tour.
- Choose the circuit of minimum distance.



Method of Trees

Total # circuits w/o Repeats

Fundamental Principle of Counting

If there are *a* ways of choosing one thing, *b* ways of choosing a second after the first is chosen,…., and *z* ways of choosing the last item after the earlier choices, then the total number of choice patterns is

a • b • c • … • z.

Examples:

Barbie Doll Clothes

10 shirts

6 skirts

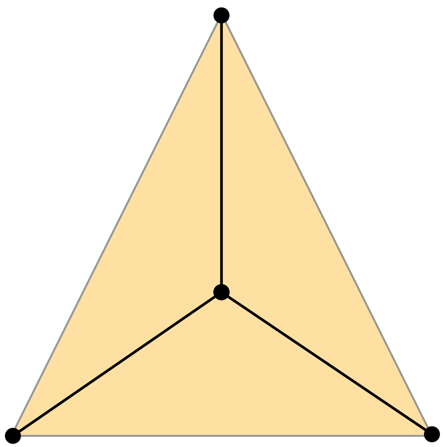
4 pairs of shoes

Drawing from a bag of 8 numbers (not replacing)

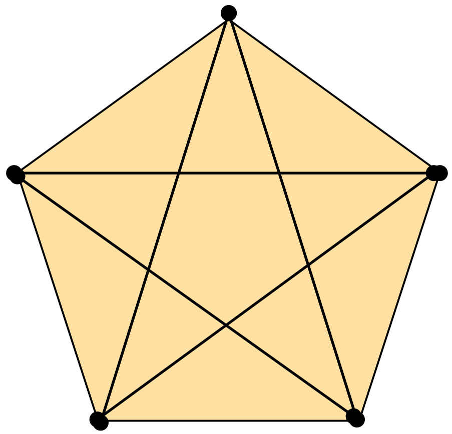
how many different orders of numbers?

Applying the Fundamental Principle of Counting

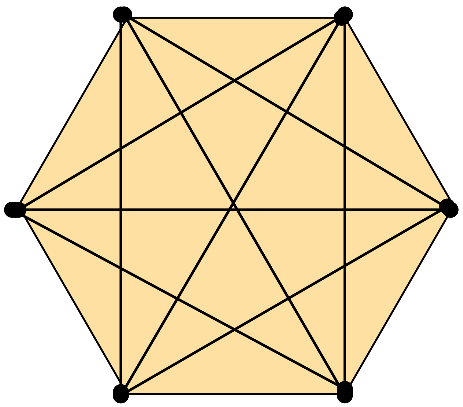
If there are *n* vertices (cities), then there are (n-1)! routes and edges. But each route is a reverse of another, so there are (n-1)!/2 different Hamiltonian Circuits or tours



four city problem



five city problem



Six city problem

four city problem	five city problem	Six city problem
Total # of routes=	Total # of routes=	Total # of routes =
Total # of tours =	Total # of tours =	Total # of tours =

Traveling Salesman Problem (TSP) textbook: pg 38-39

If the only benefit were saving money and time in vacation planning, the difficulty of finding a minimum-cost Hamiltonian circuit in a complete graph with n vertices for large values of n would not be of great concern. However, the problem we are discussing is one of the most common in operations research, the branch of mathematics concerned with getting governments and businesses to operate more efficiently. It is usually called the **traveling salesman problem (TSP)** because of its early formulation: determine the trip of minimum cost that a salesperson can make to visit the cities in a sales territory, starting and ending the trip in the same city.

Many situations require solving a TSP:

1. A lobster fisherman has set out traps at various locations and wishes to pick up his catch.
2. The telephone company wishes to pick up the coins from its pay telephone booths. (To avoid the high cost of picking up these coins, phone companies in many countries have adopted a system that uses prepurchased phone cards to operate phones. This means that there are no coins to collect!)
3. The electric (or gas) company needs to design a route for its meter readers.
4. A minibus must pick up six day campers and deliver them to camp, and later in the day return them home.
5. In drilling holes in a series of plates, the drill press operator (perhaps a robot) must drill the holes in a predetermined order.
6. Physical records, generated at automated teller machine (ATM) locations as backup in case of failure of the electronic systems, must be picked up periodically.

The meaning of cost can vary from one formulation of TSP to another. We may measure cost as distance, airplane ticket prices, time, or any other factor that is to be optimized.

In many situations, the TSP arises as a subproblem of a more complicated problem. For example, a supermarket chain may have a very large number of stores to be served from a single large warehouse. If there are fewer trucks than stores, the stores must be grouped into clusters so that one truck serves each cluster. If we then solve the TSP for every truck, we can minimize total costs for the supermarket chain. Similar vehicle-routing problems for dial-a-ride services for taking senior citizens to activity centers and for delivering children to their schools or camps often involve solving the TSP as a subproblem.

The Traveling Salesman Problem

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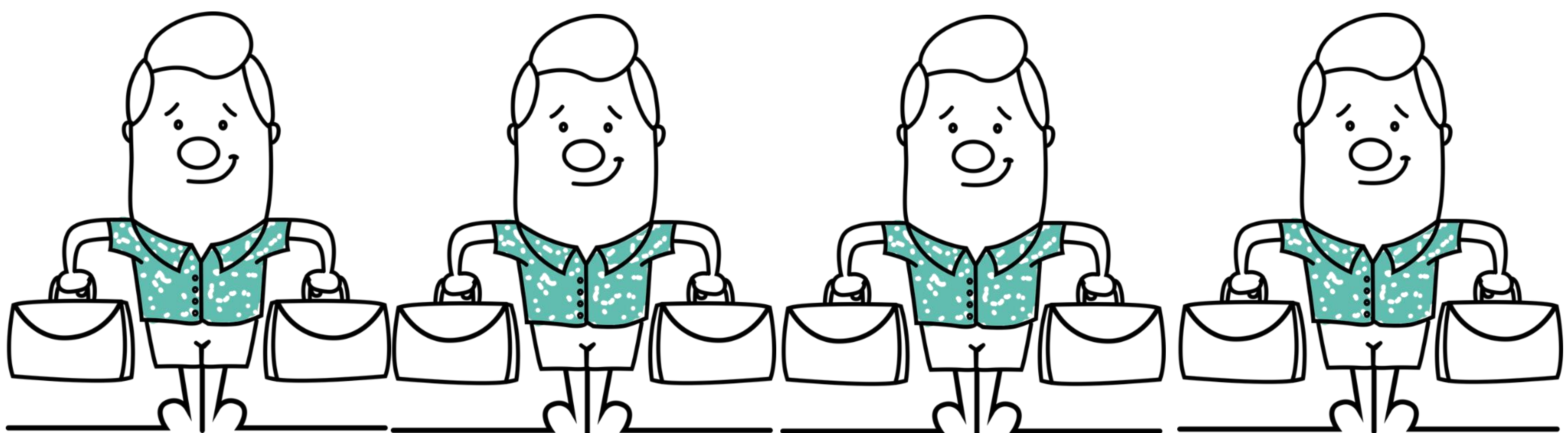

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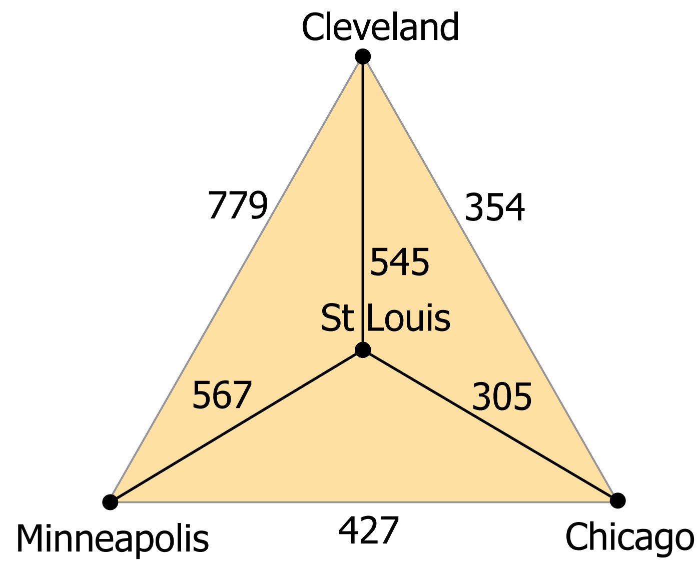
Traveling Salesman Problem (TSP)

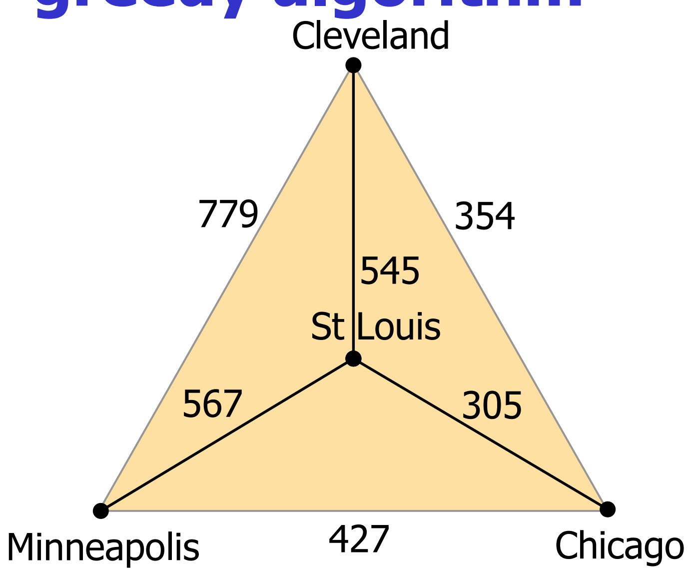
Nearest-Neighbor Algorithm

From the starting point (Chicago) first visit the nearest city, then visit the nearest city that has not already been visited. Return to the starting point when no other choice is available.



Try starting at St. Louis.

greedy algorithm

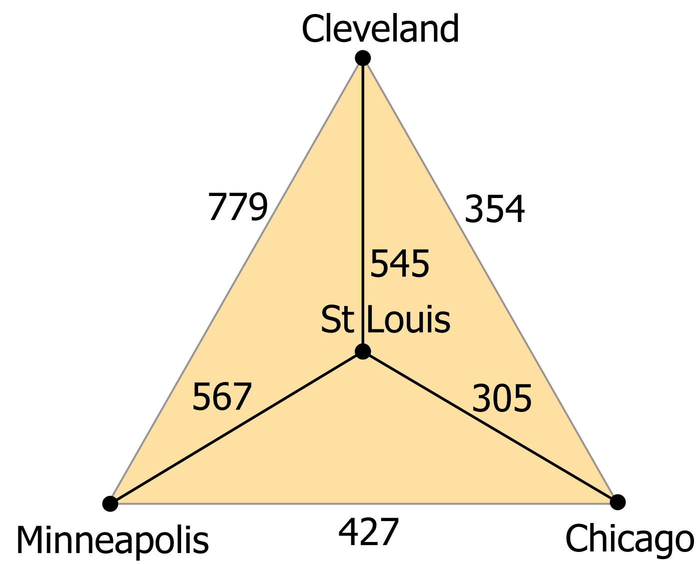


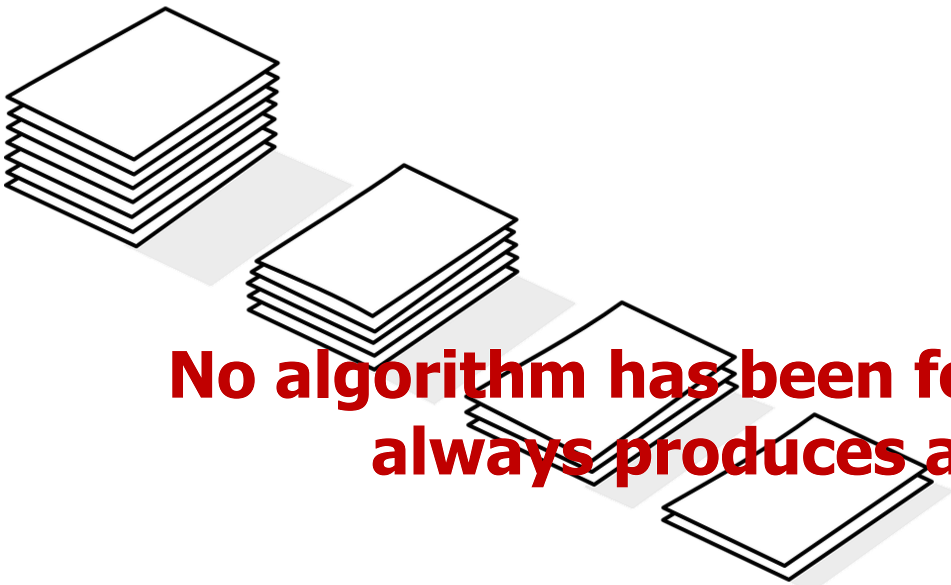
Traveling Salesman Problem (TSP)

Sorted-Edges Algorithm

Sort the edges in increasing order. Select at each stage that edge of least cost that (1) never requires that three used edges meet at a vertex and that (2) never closes up a circular tour that doesn't include all the vertices.

greedy algorithm





No algorithm has been found that is both fast and always produces an optimal solution.

Problem 1

On a map there are roads from town A of length 10, 26, 12, and 50 miles. Using the nearest-neighbor algorithm for finding a Hamiltonian circuit starting at town A, which road would be traveled first?	
A)	**road of length 10**
B)	**road of length 26**
C)	**road of length 12**
D)	**road of length 50**

Problem 2

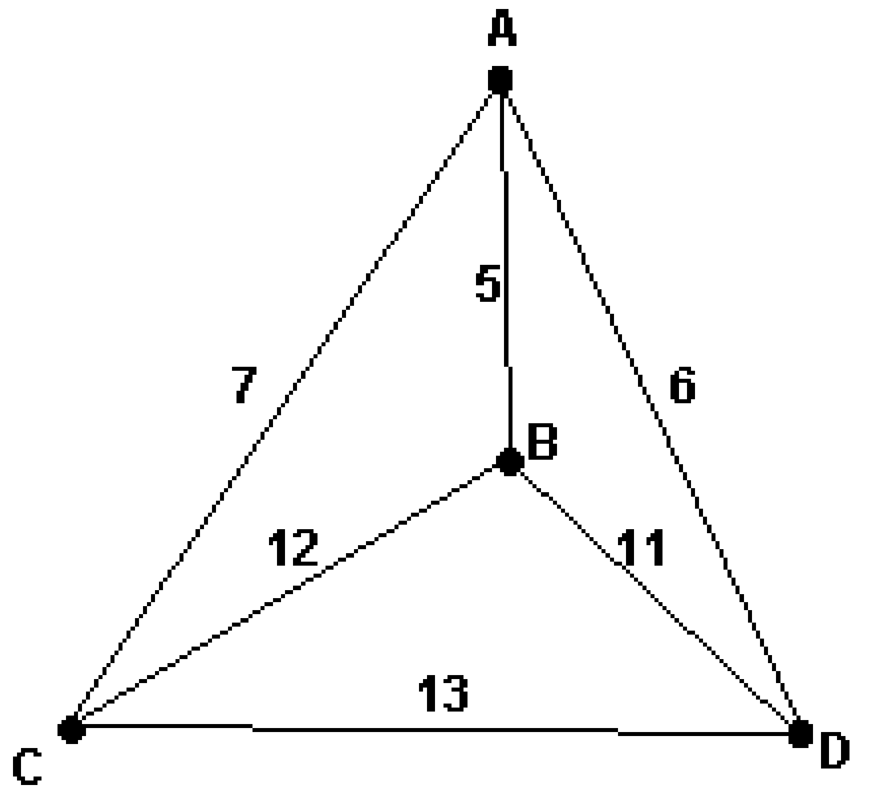
For the traveling salesman problem (Hamiltonian circuit) applied to six cities, how many tours are possible?	
A)	60
B)	120
C)	360
D)	720

Problem 3

For the traveling salesman problem (Hamiltonian circuit) applied to five cities, how many distinct tours are possible?	
A)	**120**
B)	**60**
C)	**24**
D)	**12**

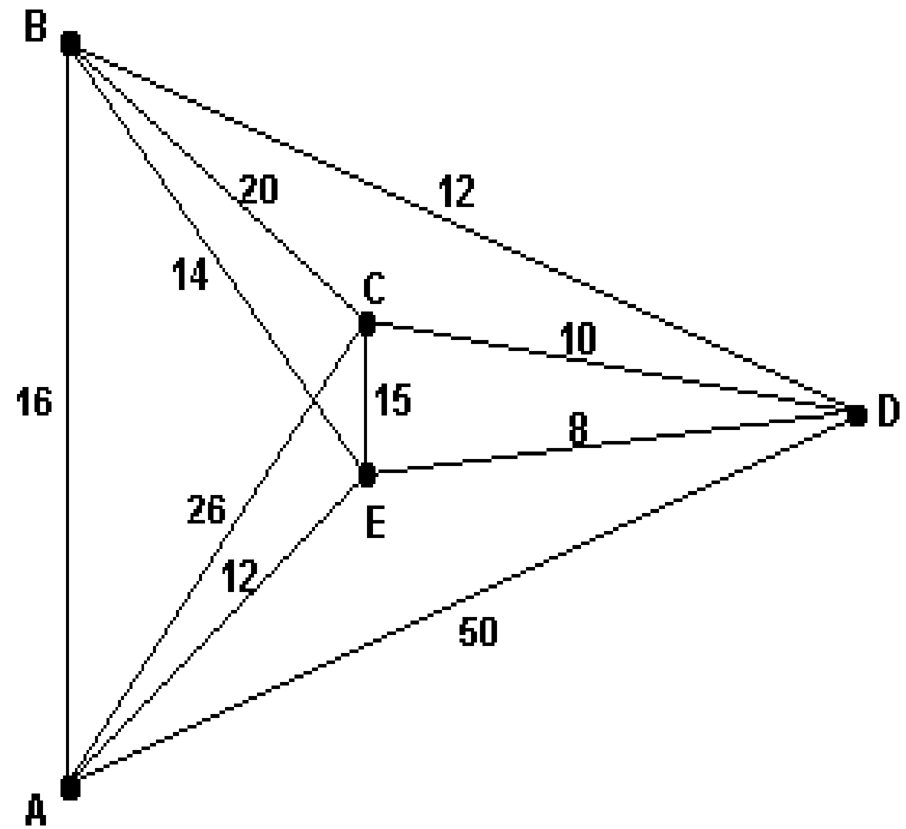
Problem 4

On the graph below, which routing is produced by using the sorted-edges algorithm to solve the traveling salesman problem?	
A)	**ABCDA**
B)	**ABDCA**
C)	**ACBDA**
D)	**ABCD**



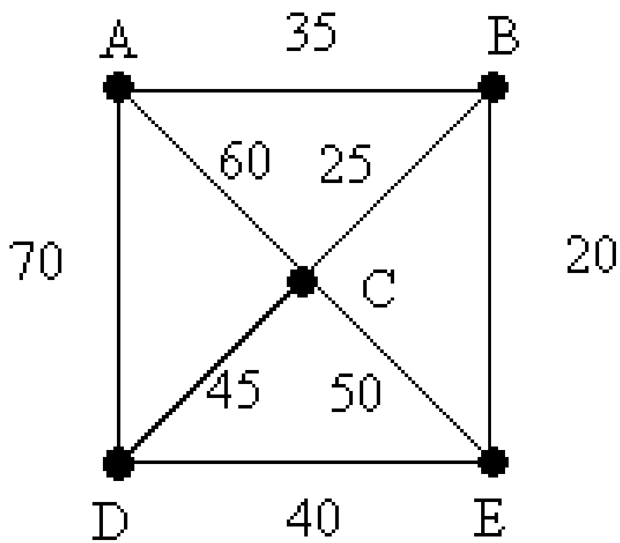
Problem 5

For the graph below, what is the cost of the Hamiltonian circuit obtained by using the sorted-edges algorithm?	
A)	40
B)	58
C)	60
D)	66



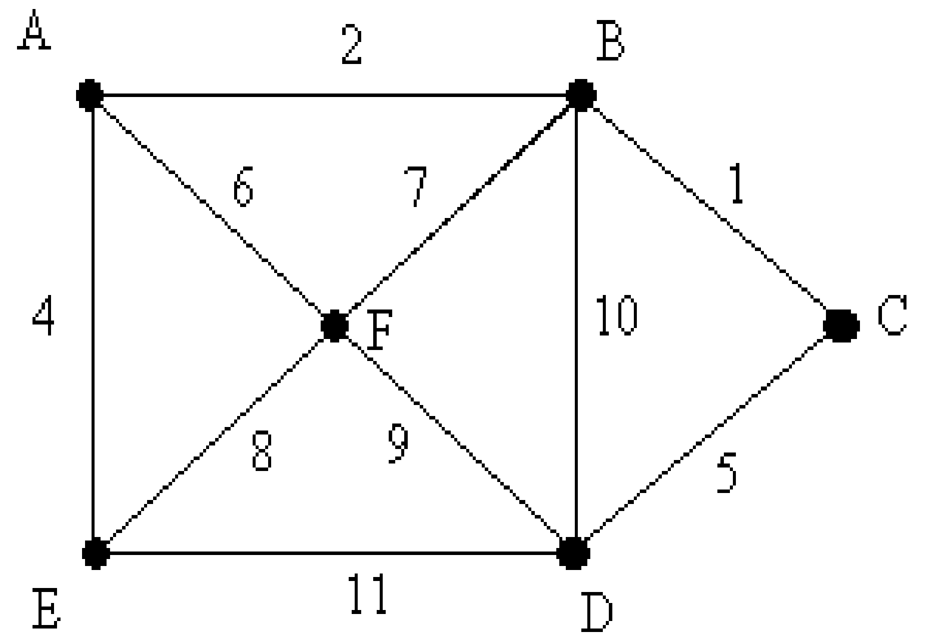
Problem 6

For the graph below, which routing is produced by using the nearest-neighbor algorithm to solve the traveling salesman problem?	
A)	ABCEDA
B)	ABEDCA
C)	ADCEBA
D)	ABCED



Problem 7

For the graph below, what is the cost of the Hamiltonian circuit obtained by using the sorted-edges algorithm?	
A)	**23**
B)	**29**
C)	**33**
D)	**41**



Problem 8

Use the brute force algorithm to solve the traveling salesman problem for the graph of the four cities shown below.