Wednesday, January 16, 2019

- Warm-up
  - In the board game Monopoly, one way to get out of jail is to roll doubles. The random variable of interest is Y=number of attempts it takes to roll doubles one time. On each roll, the probability of success is 1/6.
    - Find the probability that you roll a double within 3 turns .
    - Find the probability that it takes more than 3 turns to roll doubles, and interpret this value in context.
- Check homework
- The Binomial Model

Objectives

Content: I will use a binomial model to calculate probabilities.

Social: I will listen and participate in the class discussion.

Language: I will clarify which phrases determine a binomial model vs. those which determine a geometric model.

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The Geometric Model

- The number of trials until our first success.
- Waiting time

The Binomial Model

→ "out of" we know how many trials

- The number of successes in a given number of trials

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Binom(10, $\frac{1}{6}$)

The Binomial Model

- A Binomial model tells us the probability for a random variable that counts the number of successes in a fixed number of Bernoulli trials.
- Two parameters define the Binomial model: *n*, the number of trials; and, *p*, the probability of success. We denote this Binom(*n*, *p*).

number of trials

probability

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The Binomial Model (cont.)

- In n trials, there are

$$ {}_{n}C_{k} = \frac{n!}{k!(n-k)!} = \binom{n}{k} $$

ways to have k successes.

  - Read ${}_{n}C_{k}$ or $\binom{n}{k}$ as "*n* choose *k*."

- Note: $n! = n \times (n-1) \times \ldots \times 2 \times 1$, and $n!$ is read as "*n* factorial."

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The Binomial Model (cont.)

on formula sheet

Binomial probability model for Bernoulli trials: Binom(n,*p*)

$\mu = np$

$\sigma = \sqrt{npq}$

n = number of trials

p = probability of success

q = 1 – *p* = probability of failure

X = # of successes in *n* trials

$$P(X = x) = {}_nC_x \, p^x q^{n-x}$$

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Example

Back to the speckled M&M's. Remember that 30% of the M&M in a bag are speckled

- If I have a handful of 5 candies, how many speckled ones do I expect to get?

  $\mu = np = 5(0.3) = 1.5$ candies

  mean $E(X)$

- What is the standard deviation of the number of candies I will get?

  $\sigma = \sqrt{npq} = \sqrt{(5)(0.3)(0.7)} = 1.0247$ candies

- What is the probability that we will find 2 speckled ones in a handful of 5 candies? $P(X=2) = {}_nC_x p^x q^{n-x}$

  $P(X=2) = {}_5C_2 (0.3)^2 (0.7)^3 = \frac{5!}{2!\,3!}(0.3)^2(0.7)^3 = \frac{5 \cdot 4 \cdot 3!}{2 \cdot 3!}(0.3)^2(0.7)^3 = 10(0.3)^2(0.7)^3 = 0.308$

- What is the probability that we will find at least 2 speckled ones in a handful of 5 candies?

  $P(X=2)$ or $P(X=3)$ or $P(X=4)$ or $P(X=5) = 1 - [P(X=1) + P(X=0)]$

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binomcdf(~~5, 0.3~~, 2, 5)

"point" =

Probability
↳ Distributions

binomialpdf — one value

binomialcdf — cummulatif, multiple values → "at least" ≤ ≥ < >

$P(X=2) =$ n = 5, p = 0.3, x-value (successes) = 2 → 0.308

$P(X \text{ at least } 2) =$ n = 5 (number of trials), p = 0.3 (probability), lower bound = 2, upper bound = 5 → 0.47

Another Example

"What is the expected value or avg?" $\mu = np$

- Back to the dice rolls. What is the probability that we will roll 3 5's in a group of 20 rolls?

binomialcdf or pdf → 0.237

- What is the probability that we will roll at most 3 5's in a group of 20 rolls?

cdf lbound: 0, upbound: 3 → 0.566

- What is the probability that there will be some 5's in a group of 20 rolls?

cdf → lbound = 1, upbound = 20   0.97   1 − pdf(0)

$_{20}C_0 : \frac{20!}{0!\,20!}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{20}$ 1 · 1

- What is the probability that the first 5 is the 8th or 9th roll?

no "n" : geometric $\left(\frac{5}{6}\right)^7\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^8\left(\frac{1}{6}\right)$

Calculator “Tricks”

Scrabble

- In the game of scrabble, each player begins by drawing 7 tiles from a bag containing 100 tiles. There are 42 vowels, 56 consonants, and 2 blank tiles in the bag.
  - What is the probability of all 7 drawn being a vowel?
  - What is the probability that some of the 7 are vowels?
  - What is the probability that the vowel is the 2nd or 3rd draw?
  - How long should we expect to wait to draw a vowel?

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Objective Recheck

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Homework

- Page 403 (25, 27, 26, 30)