Tuesday, January 15, 2019

- Warm-Up

  $\mu = 0(.71) + 1(0.15) + 2(0.09) + \dots$

  $\sigma = \sqrt{(0-0.5)^2(0.71) + (1-0.5)^2(0.15) + \dots}$

  - Let X = the number of living grandparents that a randomly selected adult American has. According to recent General Social Surveys, its probability distribution is approximately P(0) = 0.71, P(1) = 0.15, P(2) = 0.09, P(3) = 0.03, P(4) = 0.02.
    - Does this refer to a discrete or continuous random variable? Why?

      Discrete - whole people

    - Find the mean and standard deviation of this probability distribution.

      $\mu = 0.5$ $\sigma = 0.92$

- Geometric Models

Objectives

Content: I will use the geometric model to determine probability and expected value.

Social: I will listen well and participate in the class discussion.

Language: I will use correct vocabulary in explaining probability situations.

Objectives

- Content Objective: I will use the geometric model to determine probability and expected value.
- Social Objective: I will listen well and participate in the class discussion.
- Language Objective: I will use correct vocabulary in explaining probability situations.

dice → success = 5
coin →

Bernoulli Trials

- The basis for the probability models we will examine in this chapter is the Bernoulli trial.
- A little background on Bernoulli
- We have Bernoulli trials if:
  - there are two possible outcomes (success and failure).
  - the probability of success, *p*, is constant.
  - the trials are independent.
- Examples…?

Objectives

Content: I will use the geometric model to determine probability and expected value.

Social: I will listen well and participate in the class discussion.

Language: I will use correct vocabulary in explaining probability situations.

The Geometric Model

Geom(p)

- A single Bernoulli trial is usually not all that interesting.
- A Geometric probability model tells us the probability for a random variable that counts the number of Bernoulli trials until the first success. "wait time"
- Geometric models are completely specified by one parameter, *p*, the probability of success, and are denoted Geom(*p*).

Objectives

Content: I will use the geometric model to determine probability and expected value.

Social: I will listen well and participate in the class discussion.

Language: I will use correct vocabulary in explaining probability situations.

The Geometric Model (cont.)

Geometric probability model for Bernoulli trials:
Geom(p)

P(X = 5)

p = probability of success

$q = 1 - p$ = probability of failure

X = number of trials until the first success occurs

$$P(X = x) = q^{x-1}p$$

$$E(X) = \mu = \frac{1}{p} \qquad \sigma = \sqrt{\frac{q}{p^2}}$$

Objectives

Content: I will use the geometric model to determine probability and expected value.

Social: I will listen well and participate in the class discussion.

Language: I will use correct vocabulary in explaining probability situations.

Example

$$E(X) = \mu = \frac{1}{p} \qquad \sigma = \sqrt{\frac{q}{p^2}}$$

- Greedy Pig
  - Waiting for a 5…
    - What are the two outcomes? 5 or not 5
    - What are their probabilities? $\frac{1}{6}$ $\frac{5}{6}$
    - Can we assume independent? How do you know? yes
    - What is our expected value? What does it mean? $1 \div \frac{1}{6} = 1 \times \frac{6}{1}$

      $E(X) = \mu = \frac{1}{\frac{1}{6}} = 6$
    - What is our standard deviation? What does it mean?

      $\sigma = \sqrt{\frac{5/6}{\left(\frac{1}{6}\right)^2}} = \sqrt{\frac{5}{6} \times \frac{36}{1}} = \sqrt{30} \approx 5.47$
    - What's the probability that the first 5 we see is the fourth roll?

      $P(X=4) = \left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)$ $\qquad P(X=x) = q^{x-1}p$
    - Simulate to see how close we are

Objectives

Content: I will use the geometric model to determine probability and expected value.

Social: I will listen well and participate in the class discussion.

Language: I will use correct vocabulary in explaining probability situations.

Another Example

- A new sales gimmick has 30% of the M&M's covered with speckles. These "groovy" candies are mixed randomly with the normal candies as they are put into the bags for distribution and sale. You buy a bag and remove candies one at a time looking for the speckles.
  - What are the two outcomes? speckles or not speckles
  - What are their probabilities? 0.3 0.7
  - Can we assume independent? How do you know? yes
  - What's the probability that the first speckled one we see is the fourth candy we get? $P(X=4) = (0.7)^3(0.3) = 0.102$
  - How many do we expect to check, on average to find a speckled one?

$$E(X) = \mu = \frac{1}{p} = \frac{1}{0.3} = 3.\overline{3}$$

$$\sigma = \sqrt{\frac{q}{p^2}} = \sqrt{\frac{0.7}{0.3^2}} \approx 2.78$$

Objectives

Content: I will use the geometric model to determine probability and expected value.

Social: I will listen well and participate in the class discussion.

Language: I will use correct vocabulary in explaining probability situations.

Sample $<$ 10% of population

Independence

10x sample $<$ population

- One of the important requirements for Bernoulli trials is that the trials be independent.
- When we don't have an infinite population, the trials are not independent. But, there is a rule that allows us to pretend we have independent trials:
  - The 10% condition: Bernoulli trials must be independent. If that assumption is violated, it is still okay to proceed as long as the sample is smaller than 10% of the population.

Objectives

Content: I will use the geometric model to determine probability and expected value.

Social: I will listen well and participate in the class discussion.

Language: I will use correct vocabulary in explaining probability situations.

Objective Check

- Content Objective: I will use the geometric model to determine probability and expected value.
- Social Objective: I will listen well and participate in the class discussion.
- Language Objective: I will use correct vocabulary in explaining probability situations.

Homework

- Page 402 (9,11,10,12, 13-14)