# Monday, January 14, 2019

- Warm-up
  - Scores on the ACT test for the 2007 high school graduating class had mean 21.2 and standard deviation 5.0. This is approximated by a Normal curve of N(21.2, 5.0). Find the z-score of a score of 27, then determine the percent of scores greater than 27.

**Objectives**

Content: I will use the mean and standard deviation of Normal distributions to determine probability of continuous variables.

Social: I will focus on the lesson so as to understand the content.

Language: I will listen carefully and use correct vocabulary in class discussion.

- Continuous Random Variables

# Objectives

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- Social Objective: I will focus on the lesson so as to understand the content.
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# Warm-up

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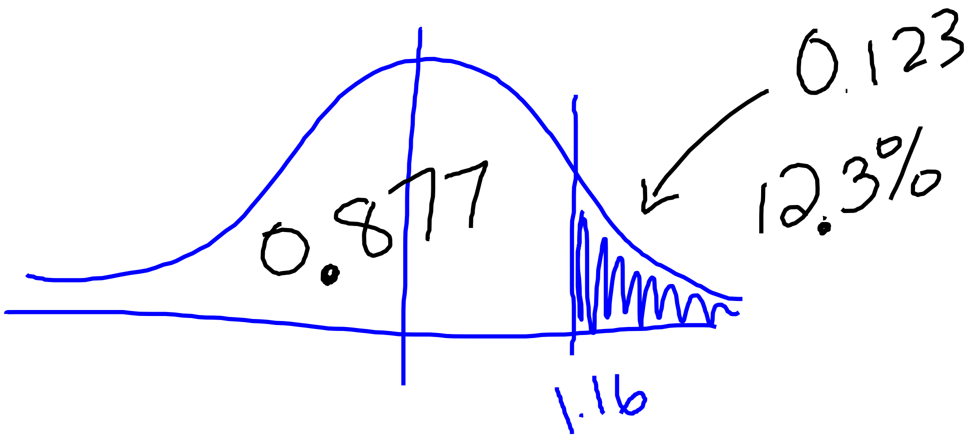
$$z = \frac{27 - 21.2}{5} = 1.16$$



- Suppose a used car dealer runs autos through a two-stage process to get them ready to sell. The mechanical checkup costs \$50 per hour and takes an average of 90 minutes, with a standard deviation of 15 minutes. The appearance prep (wash, polish, etc.) costs \$6 per hour and takes an average of 60 minutes, with a standard deviation of 5 minutes $E(M)=90$ $SD(M)=15$ $E(A)=60$ $SD(A)=5$
  - What are the mean and standard deviation of the total time spent preparing a car? $E(M+A)=E(M)+E(A)=90+60=150$ min
    $SD(M+A)=\sqrt{SD(M)^2+SD(A)^2}=\sqrt{15^2+5^2}=15.811$ min
    225 ← 25 ← variance → 250
  - What are the mean and standard deviation of the difference in time for the two phases to prepare a car? $E(M-A)=E(M)-E(A)=90-60=30$ min
    $SD(M-A)=\sqrt{SD(M)^2+SD(A)^2}=\sqrt{15^2+5^2}=15.811$ min
  - What are the mean and standard deviation of the difference in costs for the two phases of the operation?

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# Continuous Random Variables

- Random variables that can take on any value in a range of values are called continuous random variables.
- Now, any *single value* won't have a probability, but…
- Continuous random variables have means (expected values) and variances.
- We won't worry about how to calculate these means and variances in this course, but we can still work with models for continuous random variables when we're given the parameters.

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# Continuous Random Variables (cont.)

- Good news: nearly everything we've said about how discrete random variables behave is true of continuous random variables, as well.
- When two independent continuous random variables have Normal models, so does their sum or difference.
- This fact will let us apply our knowledge of Normal probabilities to questions about the sum or difference of independent random variables.

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  - What is the probability that it will take longer to do the appearance prep than the mechanical checkup? (we must assume that each process can be explained by a Normal model)

$E(M-A) = 30\text{ min}$

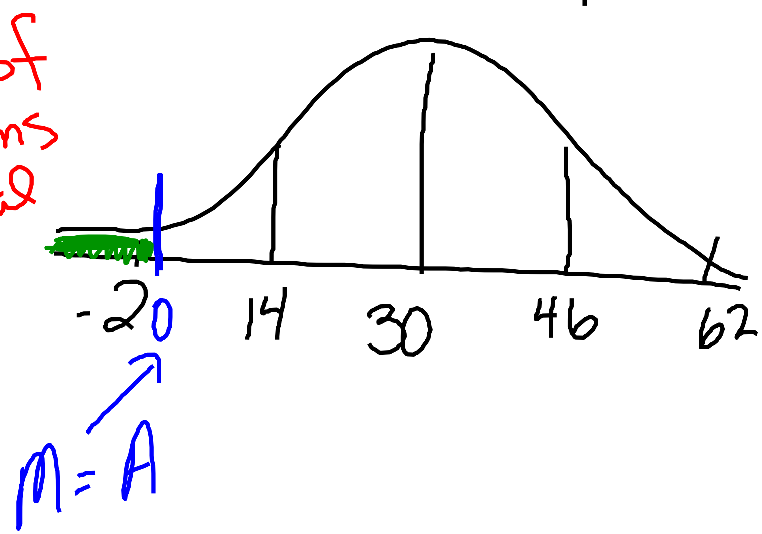
$SD(M-A) = 15.811$ (16)

Difference of zero means they are equal

$z = \frac{0-30}{16} = \frac{-30}{16} = -1.875$

0.0307

3.07%

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# Another example

$E(C) = 3.96 \quad SD(C) = 0.01 \qquad E(L) = 3.98 \quad SD(L) = 0.02$

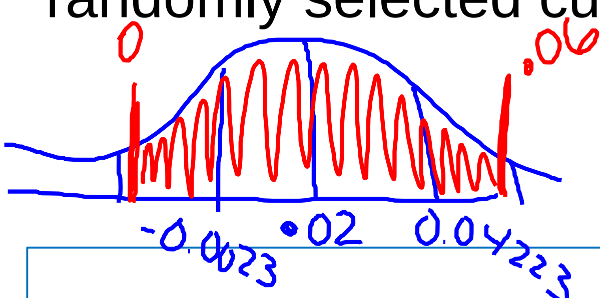
- The diameter C of a randomly selected large drink cup at a fast-food restaurant follows a Normal distribution with a mean of 3.96 inches and a standard deviation of 0.01 inches. The diameter L of a randomly selected large lid at this restaurant follows a Normal distribution with mean 3.98 inches and standard deviation 0.02 inches. For a lid to fit on a cup, the value of L has to be bigger than the value of C, but not by more than 0.06 inches.
  - How much larger to you expect the lid to be than the cup?

    $E(L-C) = E(L) - E(C) = 3.98 - 3.96 = 0.02$
  - What's the standard deviation of this difference?

    $SD(L-C) = \sqrt{SD(L)^2 + SD(C)^2} = \sqrt{0.02^2 + 0.01^2} = 0.0223$
  - What is the probability that a randomly selected lid will fit on a randomly selected cup?

    normalcdf(0, 0.06, 0.02, 0.0223)

    lbound, ubound, $\mu$, $\sigma$




0.7786

77.86%

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# Chapter Summary: What Can Go Wrong?

- Probability models are still just models.
  - Models can be useful, but they are not reality.
  - Question probabilities as you would data, and think about the assumptions behind your models.
- If the model is wrong, so is everything else.

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# What Can Go Wrong? (cont.)

- Don’t assume everything’s Normal.
  - You must *Think* about whether the Normality Assumption is justified.
- Watch out for variables that aren’t independent:
  - You can add expected values for *any* two random variables, but
  - you can only add variances of *independent* random variables.

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# What Can Go Wrong? (cont.)

- Don’t forget: Variances of independent random variables add. Standard deviations don’t.
- Don’t forget: Variances of independent random variables add, even when you’re looking at the difference between them.
- Don’t write independent instances of a random variable with notation that looks like they are the same variables.

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# What have we learned?

- We know how to work with random variables.
  - We can use a probability model for a discrete random variable to find its expected value and standard deviation.
- The mean of the sum or difference of two random variables, discrete or continuous, is just the sum or difference of their means.
- And, *for independent random variables*, the variance of their sum or difference is always the *sum* of their variances.

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# What have we learned? (cont.)

- Normal models are once again special.
  - Sums or differences of Normally distributed random variables also follow Normal models.

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# Homework

- P 385 (37, 39, 38, 40)