Friday, January 11, 2018

- Warm-up
  - Faked numbers in tax returns, invoices, or expense account claims often display patterns that aren't present in legitimate records. Some patterns, like too many round numbers, are obvious and easily avoided by a clever crook. Others are more subtle. It is a striking fact that the first digits of numbers in legitimate records often follow a model known as Benford's law. Call the first digit of a randomly chosen record X for short. Below is a **probability model** for this situation.

X	1	2	3	4	5	6	7	8	9
Prob.	0.301	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.046

  - Calculate the Expected Value (E(X) or μ)
  - Calculate the variance and standard deviation
- More with Expected Value, Variance & Standard Deviation

Objectives

Content: I will apply changes of random variables to changes in expected value, variance and standard deviation.

Social: I will participate in class discussion and not distract others or myself from the lesson.

Language: I will write clear notes that I can refer to in description of expected value, variance and standard deviation.

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Check Homework: p 383 (1-3,6,9-11,14)

1. Expected value.

a) $\mu = E(Y) = 10(0.3) + 20(0.5) + 30(0.2) = 19$

b) $\mu = E(Y) = 2(0.3) + 4(0.4) + 6(0.2) + 8(0.1) = 4.2$

2. Expected value.

a) $\mu = E(Y) = 0(0.2) + 1(0.4) + 2(0.4) = 1.2$

b) $\mu = E(Y) = 100(0.1) + 200(0.2) + 300(0.5) + 400(0.2) = 280$

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3. Pick a card, any card.

red Q ♣ A & ♣

a)

Win	\$0	\$5	\$10	\$30
P(amount won)	$\frac{26}{52}$	$\frac{13}{52}$	$\frac{12}{52}$	$\frac{1}{52}$

$+ = 1 = \frac{52}{52}$

b) $\mu = E(\text{amount won}) = \$0\left(\frac{26}{52}\right) + \$5\left(\frac{13}{52}\right) + \$10\left(\frac{12}{52}\right) + \$30\left(\frac{1}{52}\right) \approx \$4.13$

c) Answers may vary. In the long run, the expected payoff of this game is \$4.13 per play. Any amount less than \$4.13 would be a reasonable amount to pay in order to play. Your decision should depend on how long you intend to play. If you are only going to play a few times, you should risk less.

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6. Carnival.

cost $5
payout 100

10% chance

a)

Net winnings	$95	$90	$85	$80	-$20
number of darts	1 dart	2 darts	3 darts	4 darts (win)	4 darts (lose)
P(Amount won)	$\left(\frac{1}{10}\right) = 0.1$	$\left(\frac{9}{10}\right)\left(\frac{1}{10}\right) = 0.09$	$\left(\frac{9}{10}\right)^2\left(\frac{1}{10}\right) = 0.081$	$\left(\frac{9}{10}\right)^3\left(\frac{1}{10}\right) = 0.0729$	$\left(\frac{9}{10}\right)^4 = 0.6561$

1 −

b) $\mu = E(\text{number of darts}) = 1(0.1) + 2(0.09) + 3(0.081) + 4(0.0729) + 4(0.6561) \approx 3.44$ darts

c) $\mu = E(\text{winnings}) = \$95(0.1) + \$90(0.09) + \$85(0.081) + \$80(0.0729) - \$20(0.6561) \approx \$17.20$

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9. Variation 1.

a)

$$\sigma^2 = Var(Y) = (10-19)^2(0.3) + (20-19)^2(0.5) + (30-19)^2(0.2) = 49$$

actual − μ

$$\sigma = SD(Y) = \sqrt{Var(Y)} = \sqrt{49} = 7$$

b)

$$\sigma^2 = Var(Y) = (2-4.2)^2(0.3) + (4-4.2)^2(0.4) + (6-4.2)^2(0.2) + (8-4.2)^2(0.1) = 3.56$$

$$\sigma = SD(Y) = \sqrt{Var(Y)} = \sqrt{3.56} \approx 1.89$$

$z = \frac{x-\mu}{s}$

10. Variation 2.

a)

residual = obs − exp

$$\sigma^2 = Var(Y) = (0-1.2)^2(0.2) + (1-1.2)^2(0.4) + (2-1.2)^2(0.4) = 0.56$$

$$\sigma = SD(Y) = \sqrt{Var(Y)} = \sqrt{0.56} \approx 0.75$$

b)

$$\sigma^2 = Var(Y) = (100-280)^2(0.1) + (200-280)^2(0.2) + (300-280)^2(0.5) + (400-280)^2(0.2) = 7600$$

$$\sigma = SD(Y) = \sqrt{Var(Y)} = \sqrt{7600} \approx 87.18$$

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11. Pick another card.

Answers may vary slightly (due to rounding of the mean)

$$\sigma^2 = Var(\text{Won}) = (0-4.13)^2\left(\frac{26}{52}\right) + (5-4.13)^2\left(\frac{13}{52}\right)$$
$$+ (10-4.13)^2\left(\frac{12}{52}\right) + (30-4.13)^2\left(\frac{1}{52}\right) \approx 29.5396$$
$$\sigma = SD(\text{Won}) = \sqrt{Var(\text{Won})} = \sqrt{29.5396} \approx \$5.44$$

14. Darts.

$$\sigma^2 = Var(\text{Winnings}) = (95-17.20)^2(0.1) + (90-17.20)^2(0.09) + (85-17.20)^2(0.081)$$
$$+ (80-17.20)^2(0.0729) + (-20-17.20)^2(0.6561) \approx 2650.057$$
$$\sigma = SD(\text{Winnings}) = \sqrt{Var(\text{Winnings})} \approx \sqrt{2650.057} \approx \$51.48$$

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Talk about warm-up

X	1	2	3	4	5	6	7	8	9
Prob.	0.301	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.046

$$E(x) = \mu = (1)(0.301) + (2)(0.176) + \cdots$$
$$= 3.441$$

$$Var(x) = \sigma^2 = (1-3.441)^2(0.301) + (2-3.441)^2(0.176) + \cdots$$
$$= 6.060$$

$$SD(x) = \sigma = \sqrt{6.060}$$
$$\approx 2.46$$

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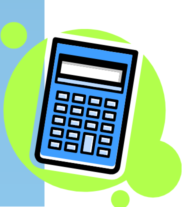

$E(x) = \mu \Rightarrow$ center Var & SD $\Rightarrow$ Spread

Remember data transformations

- How does a data shift (+ or -) affect
  - Measures of center? + or - constant
  - Measures of spread? do not change

- How does a multiplier affect
  - Measures of center? x by constant
  - Measures of spread? x

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More About Means and Variances

- Adding or subtracting a constant from data shifts the mean but doesn't change the variance or standard deviation:

$$E(X \pm c) = E(X) \pm c \qquad Var(X \pm c) = Var(X)$$

$E(X+5000) = E(x) + 5000$ $Var(X+5000) = Var(X)$

$E(x) = 2000$
$SD(x) = 1000$

  - Example: Consider everyone in a company receiving a $5000 increase in salary.

$E(x) = 7000$
$Var(x) = 1000$

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More About Means and Variances (cont.)

- In general, multiplying each value of a random variable by a constant multiplies the mean by that constant and the variance by the *square* of the constant:

$$E(aX) = aE(X) \qquad Var(aX) = a^2 Var(X)$$

$E(1.1X) = 1.1(E(x))$

$Var(1.1X) = 1.1^2 Var(x)$

$SD(1.1X) = 1.1 SD(x)$

  - Example: Consider everyone in a company receiving a 10% increase in salary. $\times 1.1$

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More About Means and Variances (cont.)

- In general,
  - The mean of the sum of two random variables is the sum of the means.
  - The mean of the difference of two random variables is the difference of the means.

$$E(X \pm Y) = E(X) \pm E(Y)$$

  - If the random variables are *independent*, the variance of their sum *or* difference is always the sum of the variances.

$$Var(X \pm Y) = Var(X) + Var(Y)$$

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Why?

- Why $Var(aX) = a^2Var(X)$ when $E(aX) = aE(X)$?
  - Remember Pythagorean Theorem?
- Why $Var(X \pm Y) = Var(X) + Var(Y)$?
  - Example
    - Grape Drink
- If the random variables are *independent*, the variance of their sum *or* difference is always the sum of the variances.

$$Var(X \pm Y) = Var(X) + Var(Y)$$

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Example

	Mean	SD	Var
X	10	2	4
Y	20	5	25

	$E(X)$	$Var(x)$	$SD(X)$
■ 3X	$E(3X) =$ $3 \times 10 = 30$	$Var(3x) =$ $3^2\, Var(x)$ $9 \times 4 = 36$	$SD(3x)$ $3 \cdot 2 =$ 6
■ Y + 6	$E(Y+6)$ $E(Y) + 6$ 26	$Var(Y+6)$ 25	5
■ X + Y	$E(X+Y) =$ $E(x)+E(Y) =$ 30	$Var(X+Y) =$ $4+25$ 29	$\sqrt{29}$
■ X – Y	$E(X-Y)$ $10-20$ -10	$Var(X-Y)$ $4+25$ 29	$\sqrt{29}$
■ $X_1 + X_2$	$E(X+X)$ $10+10$ 20	$Var(X+X)$ $4+4$ 8	$\sqrt{8}$

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Homework

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