Wednesday, January 9, 2019

- Warm-up
  - What is the probability of rolling a 5 on a dice roll? $\frac{1}{6}$
  - What is the probability that the first 5 will be your 4th roll? $\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right) = \left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right) \approx 0.096$
  - What is the probability that the first 5 will be your 10th roll? $\left(\frac{5}{6}\right)^9\left(\frac{1}{6}\right) = 0.032$
- Another Game
- Random Variables

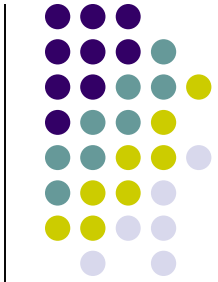
Objectives

Content: I will experiment with random variables and expected value.

Social: I will participate in class activities.

Language: I will listen for and write down key vocabulary: expected value, random variable, and the law of large numbers.

Objectives



- Content Objective: I will be able to calculate ***expected value***, ***population mean***, ***variance*** and ***standard deviation*** of a probability situation.
- Social Objective: I will participate in the class activity.
- Language Objective: I will watch for and use correct vocabulary when describing events in class today.

5+0+10+5+
10+10+0+0

Another dice game

- Consider a dice game using one regular 6 sided die to win money
  - There are no ~~points~~ money for rolling a 1, 2, or 3
  - 5 dollars for 4 or 5
  - 10 dollars for a 6
- How much would you pay to play?
- Let's play

40/8

$5

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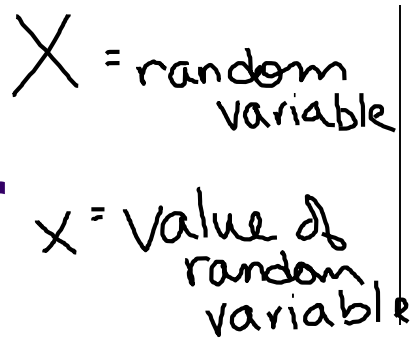
Expected Value: Center



- A random variable assumes a value based on the outcome of a random event.
  - We use a capital letter, like *X*, to denote a random variable.
  - A particular value of a random variable will be denoted with the corresponding lower case letter, in this case *x*.

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Two Types of Random Variables

- Discrete random variables can take one of a countable number of distinct outcomes.
  - Example: Shoe size
- Continuous random variables can take any numeric value within a range of values.
  - Example: Foot length

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$E(x) = \mu =$

Probability Models

- A probability model for a random variable consists of:
  - The collection of all possible values of a random variable, and
  - the probabilities that the values occur.
- Of particular interest is the value we expect a random variable to take on, notated μ (for population mean) or $E(X)$ for expected value.

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Let's Create a probability model

- Our dice game:
  - There are no points for rolling a 1, 2, or 3
  - 5 extra points for 4 or 5
  - 10 extra points for a 6

Roll	1, 2, 3	4, 5	6
Value	0	5	10
P(Value)	50% $\frac{3}{6}$ $\frac{1}{2}$	$\frac{2}{6}$ $\frac{1}{3}$	$\frac{1}{6}$

.5

Probability Model

Example: Spell Checking

Spell-checking software catches "nonword errors," which result in a string of letters that is not a word as when "the" is typed as "teh." When undergraduates are asked to write a 250-word essay (without spell-checking), the number *X* of nonword errors has the following distribution.

Value of X	0	1	2	3	4
Probability	0.1	0.2	0.3	0.3	0.1

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Using the formula

- The expected value of a (discrete) random variable can be found by summing the products of each possible value by the probability that it occurs:

$$\mu = E(X) = \sum (x \cdot P(x))$$

- *Note: Be sure that every possible outcome is included in the sum and verify that you have a valid probability model to start with.*

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Calculate Expected Value

- Our Dice game from the other day:
  - There are no points for rolling a 1, 2, or 3
  - 5 dollars for 4 or 5
  - 10 dollars for a 6

$$\mu = E(X) = \sum x \cdot P(x)$$

$$0 \cdot \frac{3}{6} + 5 \cdot \frac{2}{6} + 10 \cdot \frac{1}{6} = \frac{20}{6} \approx \$3.33$$

0 $\frac{10}{6}$ $\frac{10}{6}$

Roll	1,2,3	4,5	6
Value	0	5	10
P(Value)	3/6	2/6	1/6

Example: Spell Checking

What is the expected value (center, mean) for the number of nonword errors?

$$\mu = E(X) = \sum x \cdot P(x)$$

Value of X	0	1	2	3	4
Probability	0.1	0.2	0.3	0.3	0.1

$0 \cdot 0.1 + 1 \cdot 0.2 + 2 \cdot 0.3 + 3 \cdot 0.3 + 4 \cdot 0.1$

2.1

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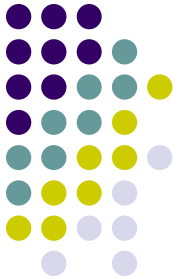
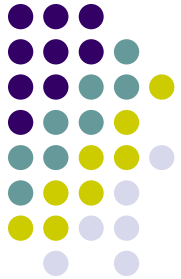
Brain Break



First Center, Now Spread…



- For data, we calculated the standard deviation by first computing the deviation from the mean and squaring it. We do that with discrete random variables as well.
- The variance for a random variable is:

$$\sigma^2 = Var(X) = \sum (x-\mu)^2 \cdot P(x)$$

- The standard deviation for a random variable is: $\sigma = SD(X) = \sqrt{Var(X)}$

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Calculate Standard Deviation

- Our Dice game:
  - There are no points for rolling a 1, 2, or 3
  - 5 dollars for 4 or 5
  - 10 dollars for a 6

$\sigma = SD(X) = \sqrt{13.88} \approx 3.72$

$$\sigma^2 = Var(X) = \sum (x-\mu)^2 \cdot P(x)$$

$$\sigma = SD(X) = \sqrt{Var(X)}$$

$\mu = 3.33$

Roll	1,2,3	4,5	6
Value	0	5	10
P(Value)	3/6	2/6	1/6

$\sigma^2 = Var(X) = (0-3.33)^2\left(\frac{3}{6}\right) + (5-3.33)^2\left(\frac{2}{6}\right) + (10-3.33)^2\left(\frac{1}{6}\right) = 13.88$

Example: Spell Checking

$\mu = 2.1$

What is the spread (variance & standard deviation) for the number of nonword errors?

$Var(x) = 1.29$ $\quad \sigma = \sqrt{1.29} = 1.13$

$$\sigma^2 = Var(X) = \sum (x-\mu)^2 \cdot P(x) \qquad \sigma = SD(X) = \sqrt{Var(X)}$$

Value of X	0	1	2	3	4
Probability	0.1	0.2	0.3	0.3	0.1

$(0-2.1)^2(0.1) + (1-2.1)^2(0.2) \ldots = 1.29$

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Homework

- P 383 (1-3, 6, 9-11, 14)