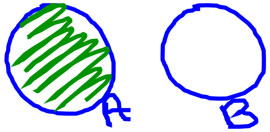
Wednesday, November 15, 2017

- Warm-up
  - The Masterfoods company says that before the introduction of purple, yellow candies made up 20% of their plain M&M's, red another 20%, and orange, blue, and green each made up 10%. The rest were brown. $\Rightarrow 30\%$
    - If you pick three M&M's in a row, what is the probability that
      - They are all brown? $= P(Br \cap Br \cap Br) = 0.3 \cdot 0.3 \cdot 0.3 = 0.027$ 2.7%
      - The third one is the first one that's red $P(NR \cap NR \cap R) = 0.8 \cdot 0.8 \cdot 0.2 = 0.128$ 12.8%
      - None are yellow? $P(NY \cap NY \cap NY) = 0.8 \cdot 0.8 \cdot 0.8 = 0.512 = 51.2\%$
      - At least one is green? $P(G \cap NG \cap NG) \cup P(G \cap G \cap G) \cup P(G \cap G \cap NG)$ $.10 \cdot .9 \cdot .9 + .1 \cdot .1 \cdot .1 + .1 \cdot .1 \cdot .9$
        $1 - P(\text{none})$
        $1 - P(NG \cap NG \cap NG)$
        $1 - (0.9 \cdot 0.9 \cdot 0.9)$
        $1 - 0.729 = \boxed{0.271}$
- Check Homework
- More with probability

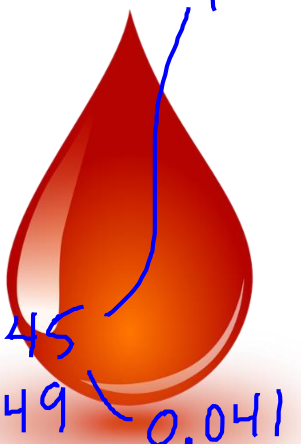
Objectives

- Content Objective: I will apply probability rules to various problems.
- Social Objective: I will participate in the activities of the class without distracting my class members.
- Language Objective: I will pay careful attention to vocabulary while listening, speaking and writing.

Practice Problems – Blood Donation

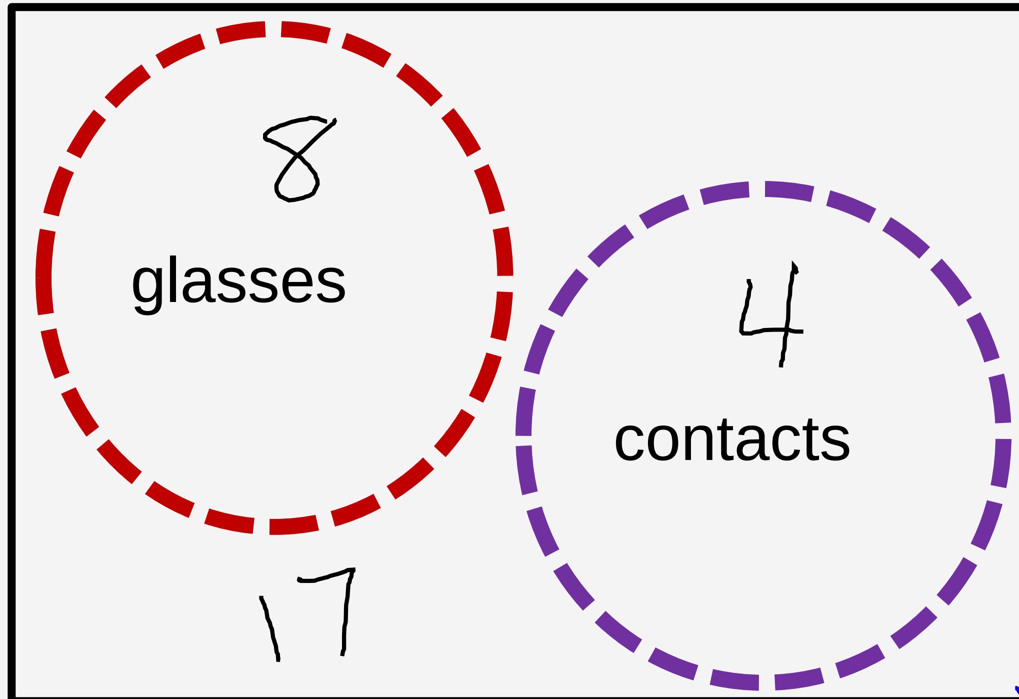


- The American Red Cross says that about 45% of the U.S. population has Type O blood, 40% Type A, 11% Type B, and the rest Type AB. 4%
  - Someone volunteers to give blood. What is the probability that this donor
    - Has Type AB blood? 4% 0.04
    - Has Type A or Type B? $.40 + .11 - 0 = 0.51$ 51% (Both)
    - Is not Type O? $1 - P(O) = 1 - 0.45 = 0.55$
  - Among four potential donors, what is the probability that
    - All are Type O? $P(O \cap O \cap O \cap O) = 0.45 \cdot 0.45 \cdot 0.45 \cdot 0.45 = (0.45)^4 = 0.041$
    - No one is Type AB? $P(AB^c \cap AB^c \cap AB^c \cap AB^c) = (0.96)^4 = 0.849$
    - They are not all Type A? $1 - P(\text{all } A) = 1 - (0.4)^4 = 1 - 0.0256 = 0.9744$
    - At least one person is Type B? $1 - P(\text{none type } B) = 1 - (0.89)^4 = 0.3725$
  - If you examine one person, are the events that the person is Type A and that the person is Type B disjoint, independent, or neither? no
  - If you examine two people, are the events that the first is Type A and the second is Type B disjoint, independent, or neither?
    Can disjoint events ever be independent?



Video to refresh...

Example 1
EYEWEAR



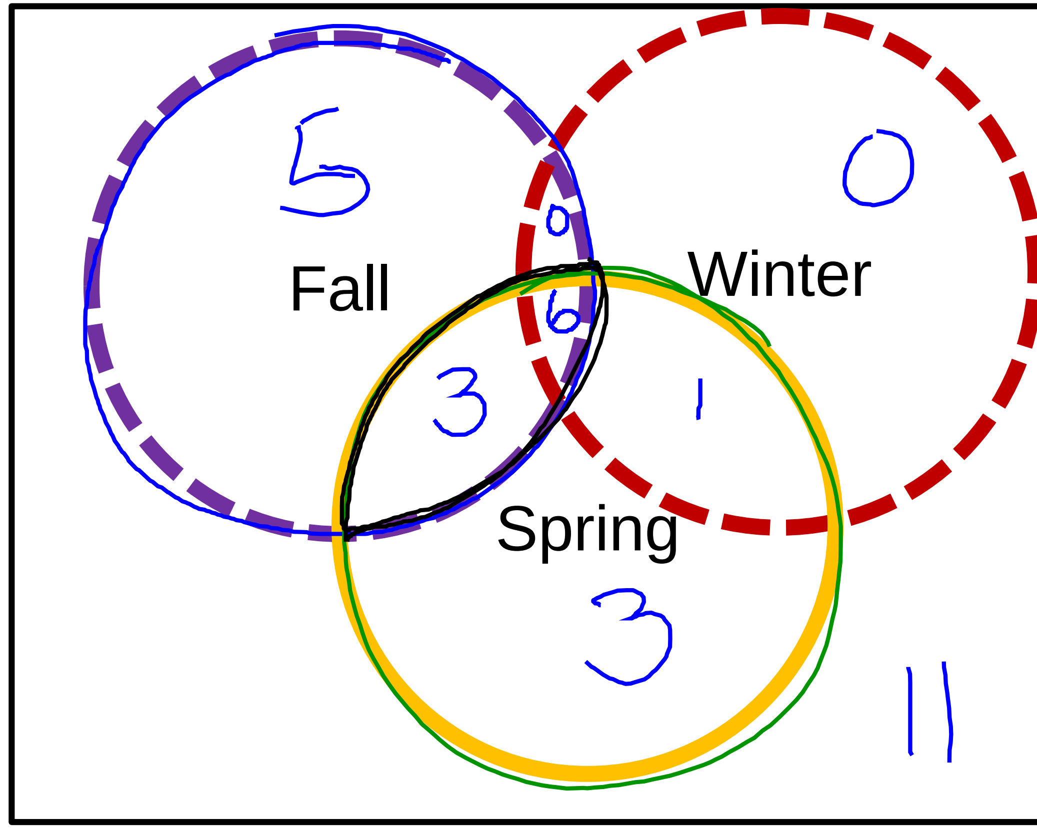
$$P(G) + P(C) - \underbrace{P(G \cap C)}_{0}$$

- P(glasses) = $\frac{8}{29}$
- P(contacts) = $\frac{4}{29}$
- P(glasses and contacts) = $\frac{0}{29}$
- P(glasses or contacts) = $\frac{12}{29}$

Example 2
SPORTS

- **P(fall) =** $\frac{14}{29}$
- **P(spring) =** $\frac{13}{29}$
- **P(fall and spring)=** $\frac{9}{29}$
- **P(fall or spring) =**

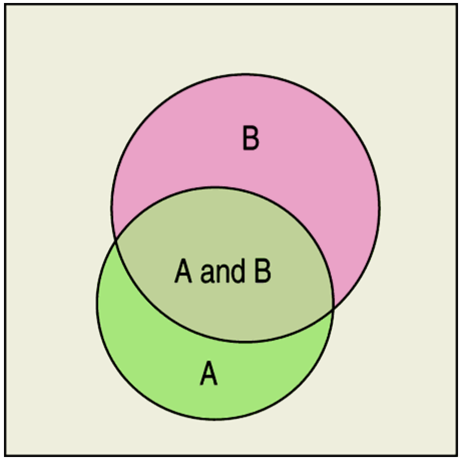
$$\frac{14}{29} + \frac{13}{29} - \frac{9}{29} = \frac{18}{29}$$



The General Addition Rule

- General Addition Rule:
  - For any two events **A** and **B**,
    $P(\mathbf{A} \cup \mathbf{B}) = P(\mathbf{A}) + P(\mathbf{B}) - P(\mathbf{A} \cap \mathbf{B})$
- The following Venn diagram shows a situation in which we would use the general addition rule:

EXAMPLE: In real estate ads it is suggested that 64% of the homes for sale have garages, 21% have swimming pools and 17% have both. What is the probability that a home has either a swimming pool or a garage?



It Depends…

- Back in Chapter 3, we looked at contingency tables and talked about conditional distributions.
- When we want the probability of an event from a *conditional* distribution, we write $P(\mathbf{B}|\mathbf{A})$ and pronounce it "the probability of **B** *given* **A**."
- A probability that takes into account a given condition is called a conditional probability.

$P(\text{male} \mid \text{tatoo}) = \frac{0}{3}$

Conditional Probability with tables

	Tattoo	No Tattoos	total
Male	0	13	13
Female	3	13	16
total	3	26	29

$\frac{13}{29} + \frac{3}{29} - \frac{0}{29} = \frac{16}{29}$

- P(male) = $\frac{13}{29}$
- P(tattoo) = $\frac{3}{29}$
- P(male and tattoo) = $\frac{0}{29}$
- P(male or tattoo) = $\frac{0}{29} + \frac{3}{29} + \frac{13}{29} = \frac{16}{29}$
- P(male | tattoo) =

It Depends... (cont.)

- To find the probability of the event **B** *given* the event **A**, we restrict our attention to the outcomes in **A**. We then find the fraction of *those* outcomes **B** that also occurred.

P(male | tatoo)

$$P(\mathbf{B}|\mathbf{A}) = \frac{P(\mathbf{A} \cap \mathbf{B})}{P(\mathbf{A})} \rightarrow \frac{P(\text{tatoo} \cap \text{male})}{P(\text{tatoo})}$$

Note: *P*(**A**) cannot equal 0, since we know that **A** has occurred.

$$\frac{0}{3}$$

EXAMPLE: P(spring|fall) =

The General Multiplication Rule (cont.)

- We encountered the general multiplication rule in the form of conditional probability.
- Rearranging the equation in the definition for conditional probability, we get the General Multiplication Rule:
  - For any two events **A** and **B**,

$$P(\mathbf{A} \cap \mathbf{B}) = P(\mathbf{A}) \times P(\mathbf{B} \mid \mathbf{A})$$

or

$$P(\mathbf{A} \cap \mathbf{B}) = P(\mathbf{B}) \times P(\mathbf{A} \mid \mathbf{B})$$

EXAMPLE: A junk box in your room contains a dozen old batteries, five of which are totally dead. You start picking batteries one a time and testing them. What is the probability that the first two you choose are good?

Independence

$P(B|A) = P(B)$

Independent

- Independence of two events means that the outcome of one event does not influence the probability of the other.

$P(\text{tatoo}|\text{male}) \stackrel{?}{=} P(\text{tatoo})$

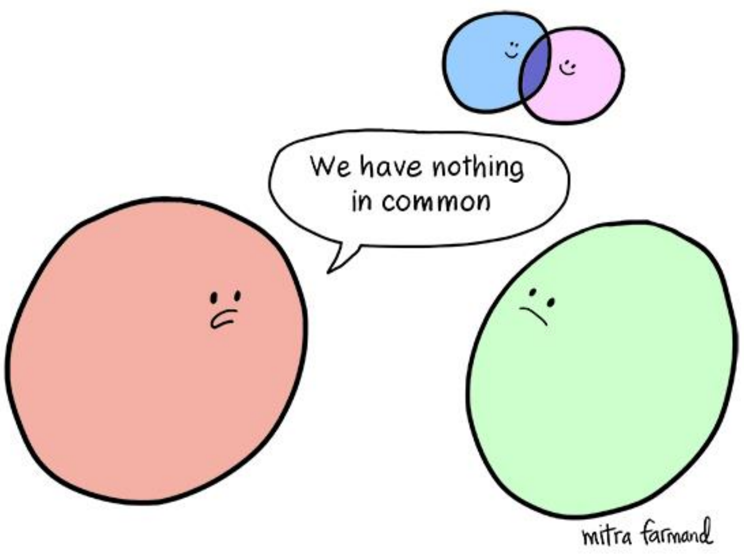
$\frac{0}{3} \stackrel{?}{=} \frac{3}{29}$ not =

not independent

- With our new notation for conditional probabilities, we can now formalize this definition:
  - Events **A** and **B** are independent whenever $P(\mathbf{B}|\mathbf{A}) = P(\mathbf{B})$. (Equivalently, events **A** and **B** are independent whenever $P(\mathbf{A}|\mathbf{B}) = P(\mathbf{A})$.)

Independent ≠ Disjoint

- Disjoint events *cannot* be independent! Well, why not?
  - Since we know that disjoint events have no outcomes in common, knowing that one occurred means the other didn't.
  - Thus, the probability of the second occurring changed based on our knowledge that the first occurred.
  - It follows, then, that the two events are *not* independent.
- A common error is to treat disjoint events as if they were independent, and apply the Multiplication Rule for independent events—don't make that mistake.



Harold had to face the painful truth. He and Daisy were never going to be a Venn diagram.

Depending on Independence

- It's much easier to think about independent events than to deal with conditional probabilities.
  - It seems that most people's natural intuition for probabilities breaks down when it comes to conditional probabilities.
- Don't fall into this trap: whenever you see probabilities multiplied together, stop and ask whether you think they are really independent.

Drawing Without Replacement

- Sampling without replacement means that once one individual is drawn it doesn't go back into the pool.
    - We often sample without replacement, which doesn't matter too much when we are dealing with a large population.
    - However, when drawing from a small population, we need to take note and adjust probabilities accordingly.
- Drawing without replacement is just another instance of working with conditional probabilities.

Homework

P 361 (2-6)