Tuesday, November 27, 2018

- Warm-up
  - Find the following probabilities using a single deck of cards and one card draw

    52

    - P(2) = $\frac{4}{52} = \frac{1}{13}$
    - P(red card) = $\frac{26}{52} = \frac{1}{2}$
    - P(red 2) = $\frac{2}{52} = \frac{1}{26}$
    - P(♣) = $\frac{13}{52} = \frac{1}{4}$
- Examples with vocabulary

Objectives

- Content Objective: I will practice simple probability with correct vocabulary.
- Social Objective: I will be respectful and participate in class examples.
- Language Objective: I will ask for clarification on vocabulary that I do not understand and use the correct vocabulary in questions and explanations.

Modeling Probability

- When probability was first studied, a group of French mathematicians looked at games of chance in which all the possible outcomes were *equally likely*. They developed mathematical models of **theoretical probability**.
  - It's equally likely to get any one of six outcomes from the roll of a fair die.
  - It's equally likely to get heads or tails from the toss of a fair coin.
- However, keep in mind that events are *not* always equally likely.
  - A skilled basketball player has a better than 50-50 chance of making a free throw.

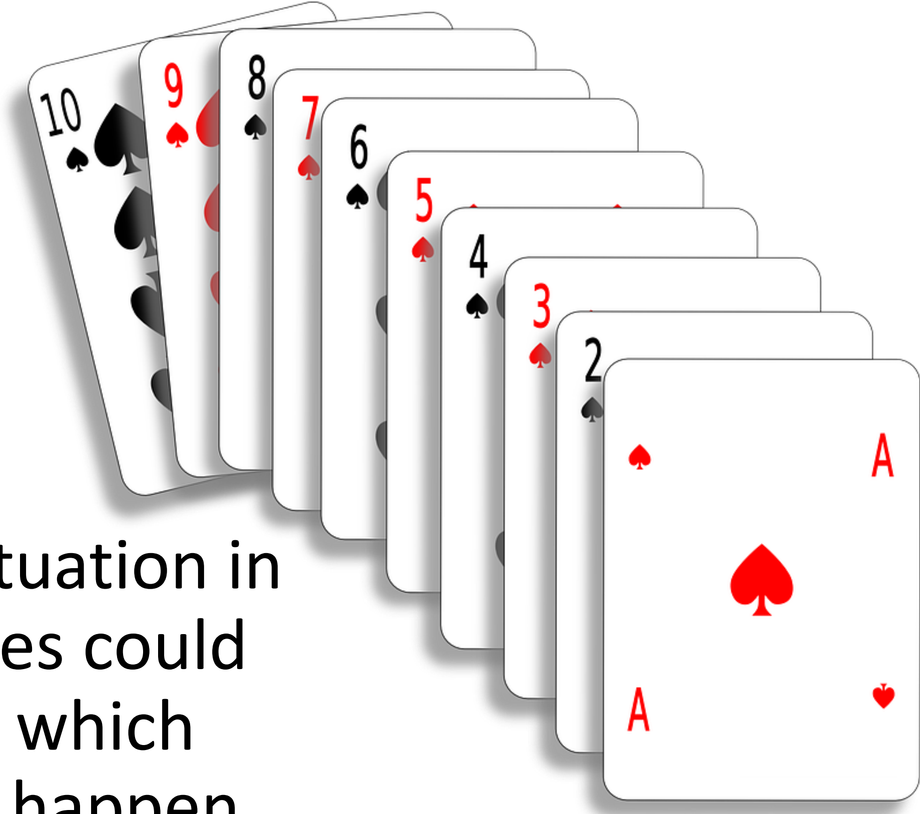
Modeling Probability

$P(2) = \frac{4}{52}$

The probability of an event is the number of outcomes in the event divided by the total number of possible outcomes.

$$\boxed{P(A) = \frac{\text{\# of outcomes in A}}{\text{\# of possible outcomes}}}$$

Dealing with Random Phenomena



A **random phenomenon** is a situation in which we know what outcomes could happen, but we don't know which particular outcome did or will happen.

Dealing with Random Phenomena

In general, each occasion upon which we observe a random phenomenon is called a **trial**.

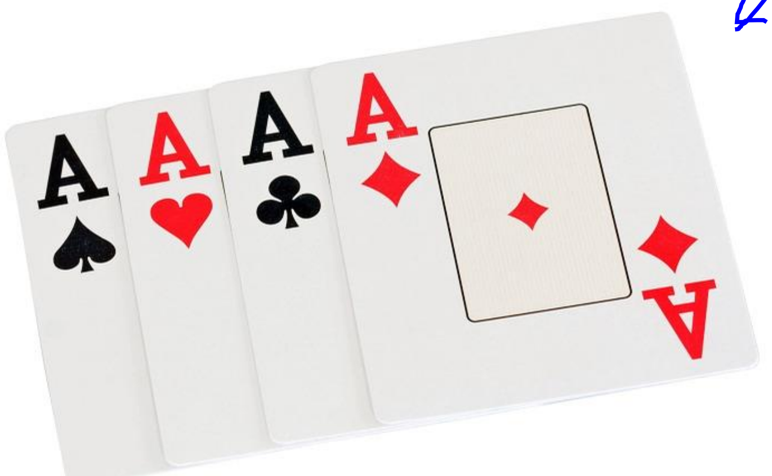
Dealing with Random Phenomena

At each trial, we note the value of the random phenomenon, and call it an **outcome**. result

outcome - A

Dealing with Random Phenomena

When we combine outcomes, the resulting combination is an **event**.



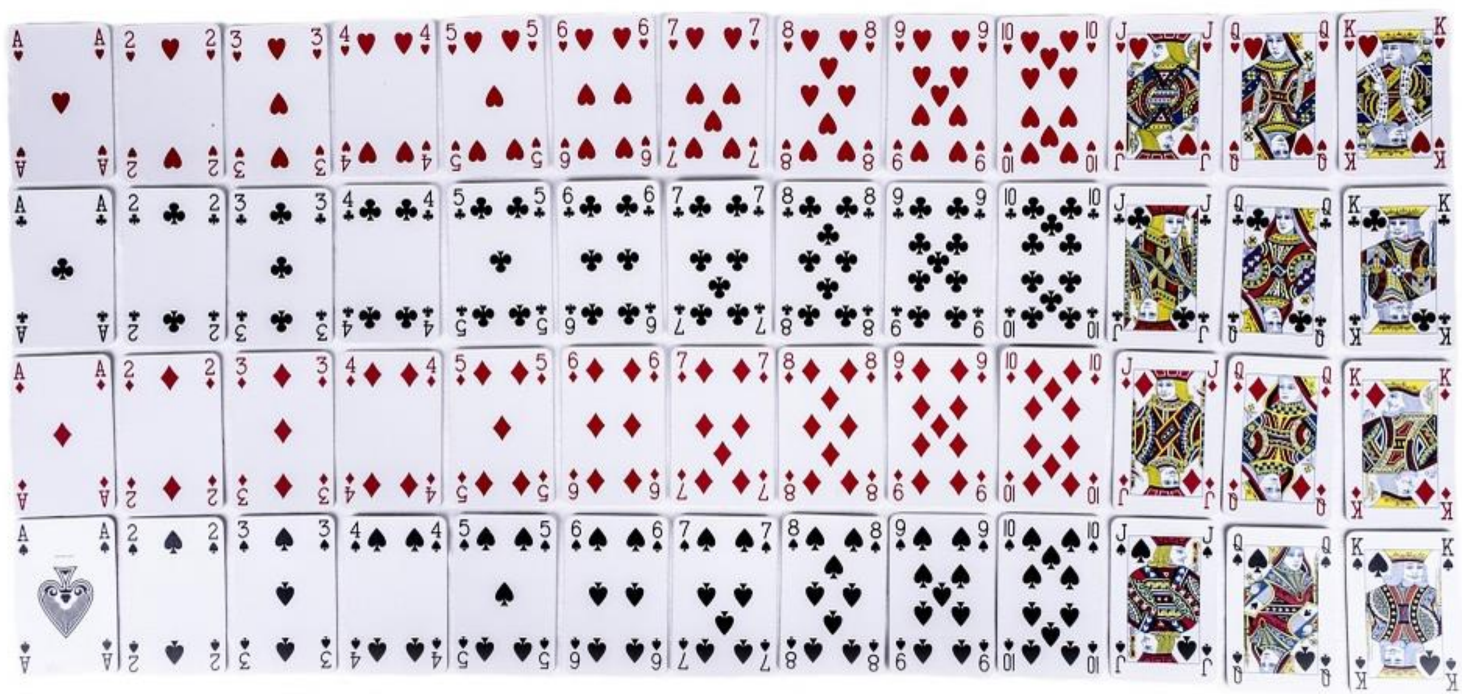
Dealing with Random Phenomena

1, 2, 3, 4, 5, 6

2 . . .

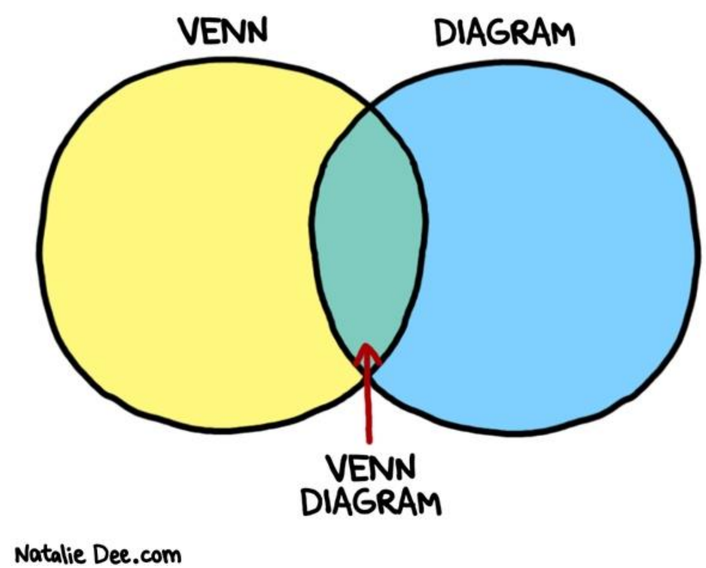
, 2

- The collection of *all possible outcomes* is called the **sample space**.

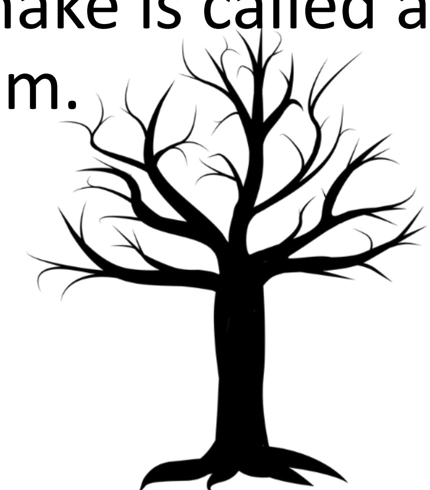


The First Three Rules of Working with Probability

- We are dealing with probabilities now, not data, but the three rules don't change.
  - Make a picture.
  - Make a picture.
  - Make a picture.



- The most common kind of picture to make is called a Venn diagram.



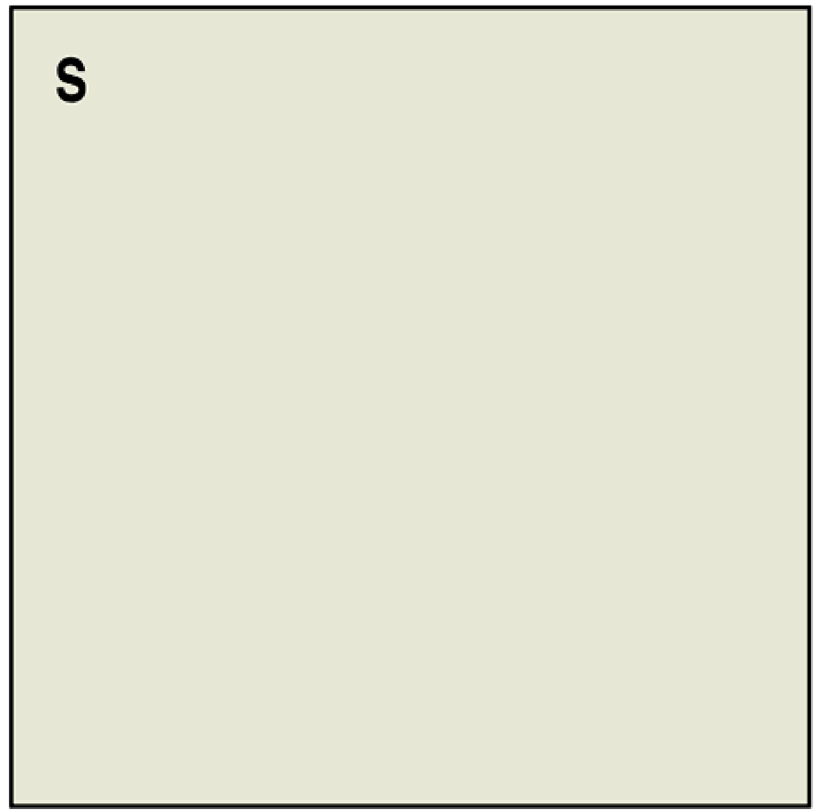
- We will also be making tree diagrams

Formal Probability

1. Two requirements for a probability:
   - A probability is a number between 0 and 1.
   - For any event **A**, $0 \leq P(\mathbf{A}) \leq 1$.

2. Probability Assignment Rule:

- The probability of the set of all possible outcomes of a trial must be 1.
- $P(\mathbf{S}) = 1$ (**S** represents the set of all possible outcomes.)



The sample space **S.**

3. Complement Rule:

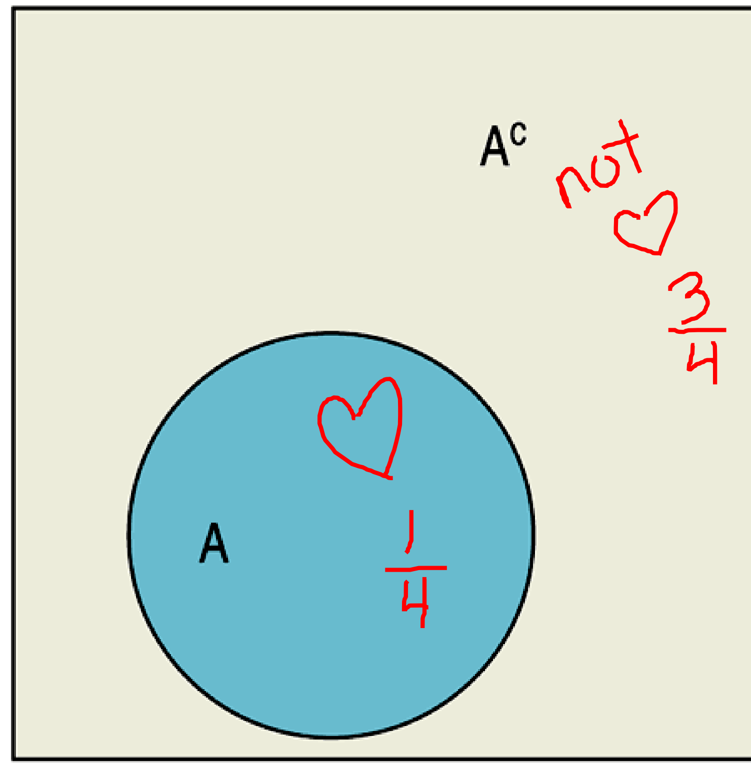
- The set of outcomes that are *not* in the event **A** is called the complement of **A**, denoted $\boxed{\mathbf{A}^{c}.}$
- The probability of an event occurring is 1 minus the probability that it doesn't occur: $P(\mathbf{A}) = 1 - P(\mathbf{A}^{c})$

$P(A^c)$

$P(\text{Black}) = \frac{4}{14}$

EXAMPLE: A bag contains 6 red marbles, 4 yellow marbles, and 4 black marbles. What is the probability that you draw a non-black marble?

$P(\text{not Black}) = \frac{10}{14}$



The set **A** and its complement.

4. Mutually Exclusive

♡ or Q

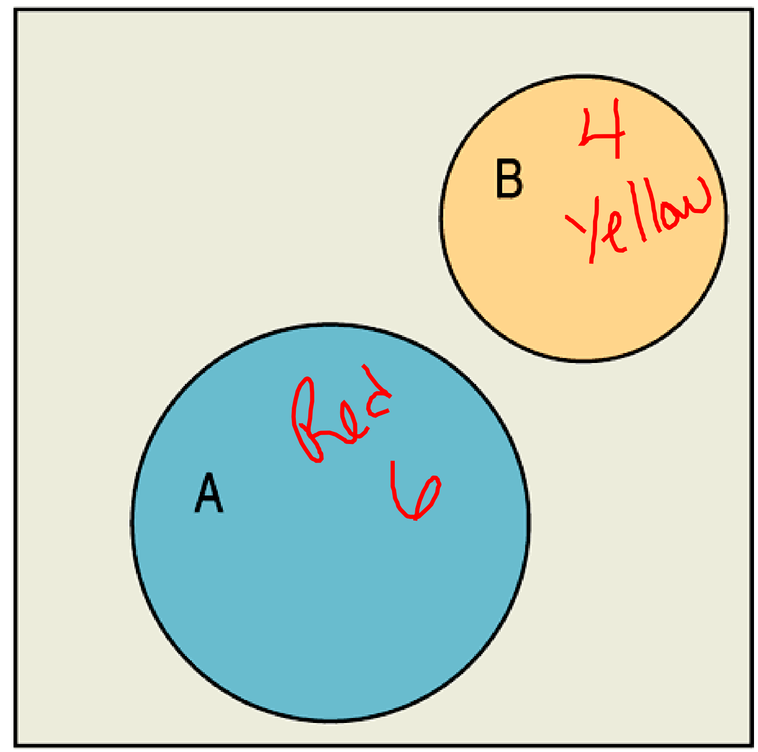
- Events that have no outcomes in common (and, thus, cannot occur together) are called disjoint (or mutually exclusive).

not ♡ or 2

♡ or red

$\frac{10}{14}$

EXAMPLE: A bag contains 6 red marbles, 4 yellow marbles, and 4 black marbles. What is the probability that you draw a yellow marble or a red marble?



Two disjoint sets, **A** and **B**.

5. General Addition Rule (cont.):

- For two events **A** and **B**, the probability that one *or* the other occurs is the sum of the probabilities of the two events minus the probability they both happen.
- $P(\mathbf{A} \cup \mathbf{B}) = P(\mathbf{A}) + P(\mathbf{B}) - P(A \cap B)$

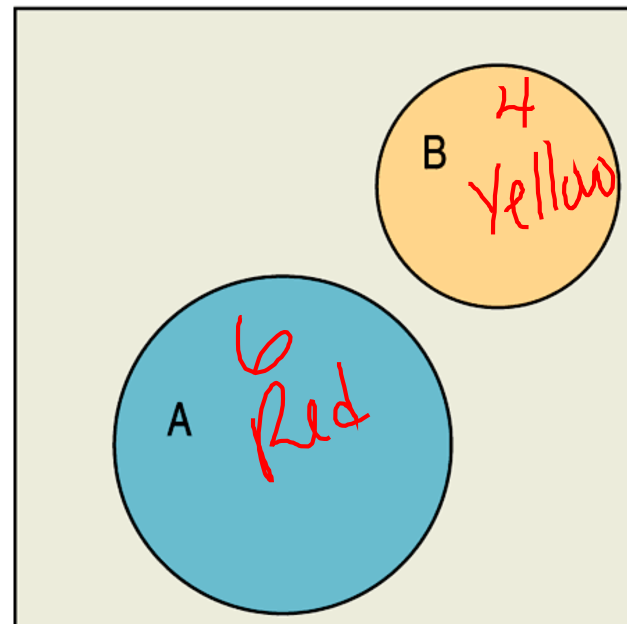
A

OR

AND

$$\frac{6}{14} + \frac{4}{14} - \frac{0}{14}$$

EXAMPLE: A bag contains 6 red marbles, 4 yellow marbles, and 4 black marbles. What is the probability that you draw a yellow marble or a red marble?



6. General Multiplication Rule:

without replacement

- For two events **A** and **B**, the probability that *both* **A** and **B** occur is the product of the probabilities of the two events.

$P(B \cap B) = \frac{4}{14} \times \frac{3}{13}$

- $P(\mathbf{A} \cap \mathbf{B}) = P(\mathbf{A}) \times P(\mathbf{B} | \mathbf{A})$

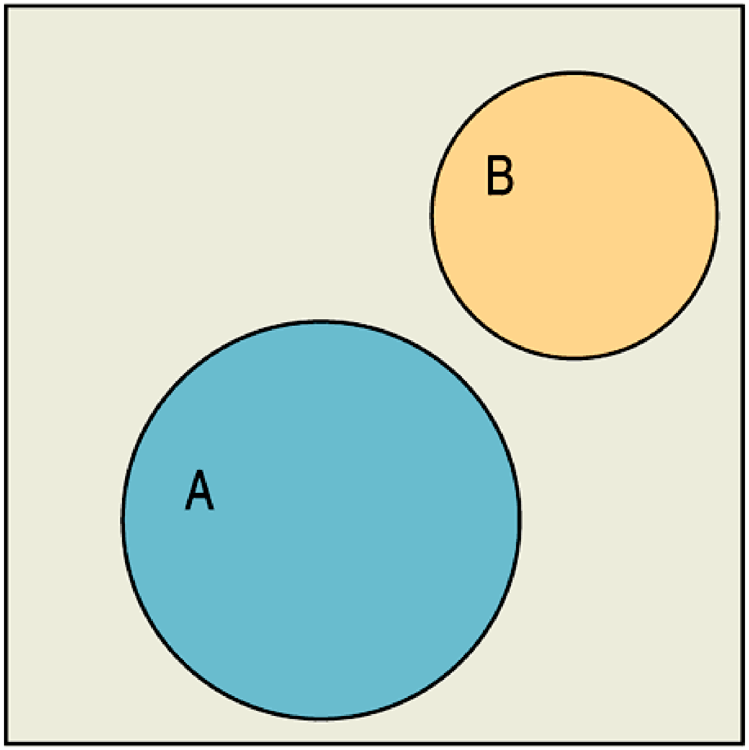
AND

B given A

$P(\text{Black} \cap \text{Black}) = \frac{4}{14} \times \frac{4}{14}$

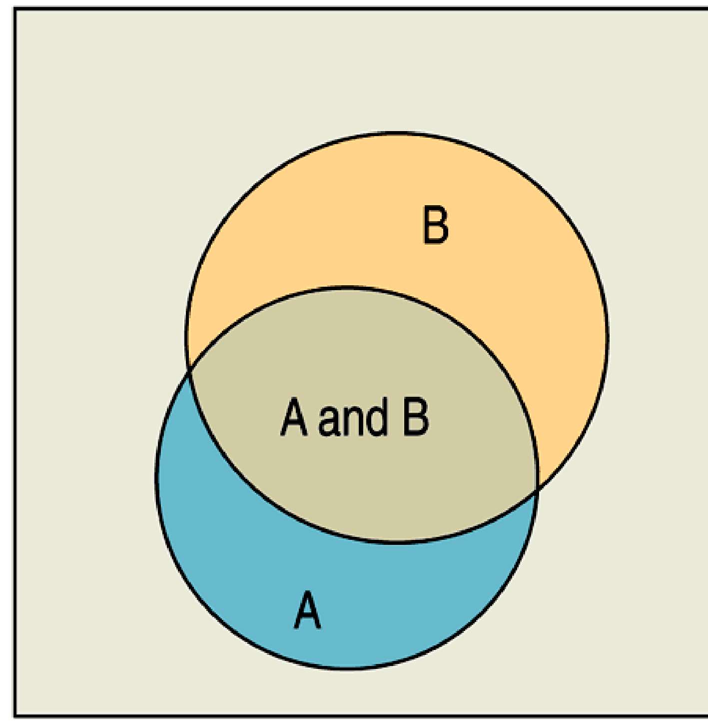
$= \frac{16}{196}$

EXAMPLE: A bag contains 6 red marbles, 4 yellow marbles and 4 black marbles. Two marbles are drawn *with replacement* from the urn. What is the probability that both of the marbles are black?



6. Multiplication Rule (cont.):

- Two independent events **A** and **B** are not disjoint, provided the two events have probabilities greater than zero:



Two sets **A** and **B** that are not disjoint. The event (**A** and **B**) is their intersection.

Notation alert:

- In this text we use the notation $P(\mathbf{A} \cup \mathbf{B})$ and $P(\mathbf{A} \cap \mathbf{B})$.
- In other situations, you might see the following:
  - $P(\mathbf{A} \text{ or } \mathbf{B})$ instead of $P(\mathbf{A} \cup \mathbf{B})$
  - $P(\mathbf{A} \text{ and } \mathbf{B})$ instead of $P(\mathbf{A} \cap \mathbf{B})$

Suppose that 40% of cars in your area are manufactured in the United States, 30% in Japan, 10% in Germany, and 20% in other countries. If the cars are selected at random, find the probability that

$$\begin{array}{r} 0.3 \\ 0.3 \\ \hline 0.09 \end{array}$$

- A car is not U.S. made $P(\text{not US}) = 1 - P(\text{U.S.})$ $= 1 - 0.4$ $= 0.6$

Complementary events

Complement Rule

- It is made in Japan or Germany $P(J \cup G)$ $= 0.3 + 0.10 - 0$ $= 0.4$

Mutually Exclusive

Disjoint Events

Addition Rule

U = union

- You see two in a row from Japan $P(J \cap J)$ $= 0.3 \cdot 0.3$ $= 0.09$

Independent Events

Multiplication rule

Suppose that 40% of cars in your area are manufactured in the United States, 30% in Japan, 10% in Germany, and 20% in other countries. If the cars are selected at random, find the probability that

- None of three cars came from Germany

$P(\text{not } G \cap \text{not } G \cap \text{not } G)$

$0.9 \times 0.9 \times 0.9 = 0.729$

- At least one of three cars is U.S. - made

none are not US made

$1 - P(\text{none of } 3) = 1 - P(\text{not US} \cap \text{not US} \cap \text{not US})$

$1 - 0.6 \times 0.6 \times 0.6 = 1 - 0.216 = 0.784$

- The first Japanese car is the fourth one you see

$P(\text{not } J \cap \text{not } J \cap \text{not } J \cap J) = 0.7 \times 0.7 \times 0.7 \times 0.3$

$= 0.102$

Independent Events
Multiplication rule
Complement Rule

Independent Events
Multiplication rule
Complement Rule

Independent Events
Multiplication rule
Complement Rule

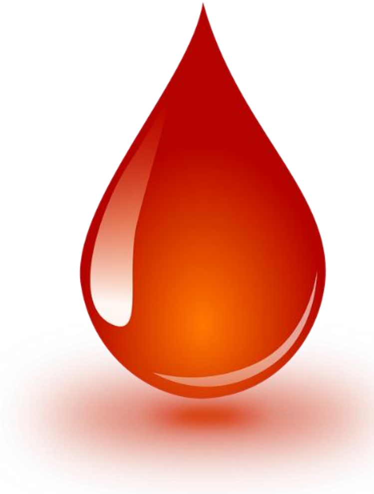
∩ = intersection

Putting the Rules to Work

- In most situations where we want to find a probability, we'll use the rules in combination.
- A good thing to remember is that it can be easier to work with the *complement* of the event we're really interested in.

Practice Problems – Blood Donation

- The American Red Cross says that about 45% of the U.S. population has Type O blood, 40% Type A, 11% Type B, and the rest Type AB.
  - Someone volunteers to give blood. What is the probability that this donor
    - Has Type AB blood?
    - Has Type A or Type B?
    - Is not Type O?
  - Among four potential donors, what is the probability that
    - All are Type O?
    - No one is Type AB?
    - They are not all Type A?
    - At least one person is Type B?
  - If you examine one person, are the events that the person is Type A and that the person is Type B disjoint, independent, or neither?
  - If you examine two people, are the events that the first is Type A and the second is Type B disjoint, independent, or neither?
    Can disjoint events ever be independent?



Homework

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