

Monday, April 23, 2018

Sample < 10%
population

- Warm-up
 - For the following multiple choice questions choose the correct answer AND explain why each incorrect answer is incorrect.

When using a one-sample t -procedure to construct a confidence interval for the mean of a finite population, a condition is that the population size be at least 10 times the sample size. The reason for the condition is to ensure that

- (A) the sample size is large enough *opposite mixing up with another condition success/fail*
- (B) the central limit theorem is applicable for the sample mean *CLT large enough = 30*
- (C) the sample standard deviation is a good approximation of the population standard deviation
- (D) the degree of dependence among observations is negligible *- replacement*
- (E) the sampling method is not biased

- Go over Friday's FRAPPY
- Work on project

2010 #5

A large pet store buys the identical species of adult tropical fish from two different suppliers—Buy-Rite Pets and Fish Friends. Several of the managers at the pet store suspect that the lengths of the fish from Fish Friends are consistently greater than the lengths of the fish from Buy-Rite Pets. Random samples of 8 adult fish of the species from Buy-Rite Pets and 10 adult fish of the same species from Fish Friends were selected and the lengths of the fish, in inches, were recorded, as shown in the table below.

	Length of Fish	Mean	Standard Deviation
Buy-Rite Pets ($n_B = 8$)	3.4 2.7 3.3 4.1 3.5 3.4 3.0 3.8	3.40	0.434
Fish Friends ($n_F = 10$)	3.3 2.9 4.2 3.1 4.2 4.0 3.4 3.2 3.7 2.6	3.46	0.550

Do the data provide convincing evidence that the mean length of the adult fish of the species from Fish Friends is greater than the mean length of the adult fish of the same species from Buy-Rite Pets?

Intent of Question

The primary goal of this question was to assess students' ability to set up, perform and interpret the results of a significance test. More specific goals were to assess students' ability to (1) state appropriate hypotheses; (2) identify the name of an appropriate statistical test and check appropriate assumptions/conditions; (3) compute the appropriate test statistic and p-value; (4) draw an appropriate conclusion, with justification, in the context of the study.

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Step 1: States a correct pair of hypotheses

Let μ_B represent the population mean length of all adult fish of this species from Buy-Rite Pets, and let μ_F represent the population mean length of all adult fish of this species from Fish Friends.

μ

The hypotheses to be tested are $H_0: \mu_B = \mu_F$ versus $H_a: \mu_B < \mu_F$.

Step 1 is scored as follows:

- Essentially correct (E) if the student uses correct parameters *AND* states correct hypotheses.
- Partially correct (P) if the student uses correct parameters *OR* states correct hypotheses but not both.
- Incorrect (I) otherwise.

2010 #5

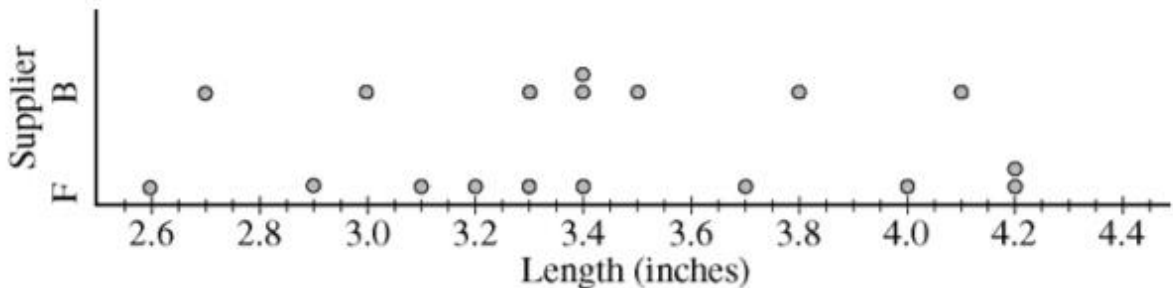
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The appropriate test is a two-sample t -test. The first condition is that the samples are independent random samples from the two populations. This was stated in the question. The second condition is that the population distributions of fish lengths are normal. The following dotplots reveal no obvious departures from normality, so it appears reasonable to proceed with the two-sample t -test.

norm. dist. Plct
box hist dist



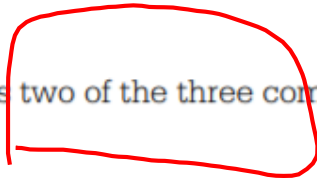
w/ diagram
nearly normal

Step 2 is scored as follows:

- Essentially correct (E) if the student correctly completes all three of the following components:
- Identifies the correct test procedure (by name or by formula)
 - Checks for independent random samples
 - Checks for normality

Partially correct (P) if the student correctly completes two of the three components listed above.

Incorrect (I) otherwise.



2010 #5

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Step 3: Demonstrates correct mechanics, including the value of the test statistic, df and p-value (or rejection region)

The test statistic is: $t = \frac{\bar{x}_B - \bar{x}_F}{\sqrt{\frac{s_B^2}{n_B} + \frac{s_F^2}{n_F}}} = \frac{3.40 - 3.46}{\sqrt{\frac{0.434^2}{8} + \frac{0.550^2}{10}}} = -0.259$

With df = 15.99999 (p-value = 0.3996.)

$z_{15.9} = -0.259$

both for test statistic

Step 3 is scored as follows:

Essentially correct (E) if the student correctly calculates both the test statistic and p-value.

Partially correct (P) if the student correctly calculates the test statistic but not the p-value OR omits the test statistic but correctly calculates the p-value.

Incorrect (I) otherwise.

2010 #5

$E = 1$
 $P = \frac{1}{2}$
 $I = 0$

add
 $3\frac{1}{2} = 3$
 $2\frac{1}{2} = ?$
 $1\frac{1}{2} = ?$
 $\frac{1}{2} = 0$

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Step 4: States a correct conclusion in the context of the problem, using the result of the statistical test

Because this p-value is larger than any conventional significance level (such as $\alpha = 0.10$ or $\alpha = 0.05$), we fail to reject H_0 . The sample data do not provide convincing evidence to conclude that the mean length of the adult fish of the species from Fish Friends is greater than the mean length of the adult fish of the same species from Buy-Rite Pets.

Step 4 is scored as follows:

Essentially correct (E) if the student provides a correct conclusion in context, also providing justification based on linkage between the p-value and conclusion.

Partially correct (P) if the student provides a correct conclusion, with linkage to the p-value, but not in context OR provides a correct conclusion in context, but without justification based on linkage to the p-value.

Incorrect (I) otherwise.

Project Work

